Fiber non-Turing all-optical computer for solving complex decision problems

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Abstract: We demonstrate an all-optical computer that solves one of the most difficult complexity problems, the Hamiltonian challenge of finding if a map can be travelled in a way that each town is visited exactly once.

There is a class of diverse mathematical complexity problems such as the travelling salesman problem of looking for a shortest possible route on a map, often referred as Nondeterministic Polynomial (NP) complete problems. There is still yet no efficient algorithm to solve these problems within polynomial time by a deterministic Turing machine. Besides the conventional approach with electronic computers, Non-Turing approaches such as DNA, quantum and optical computing may work. Here we show that these NP-complete problems may be solved by using a new type of all-optical computer. A proof-of-principle demonstration was performed on a fiber network representing a map of five towns on the NP-complete directed Hamiltonian path problem of deciding if a map can be travelled in a unidirectional way that each town is visited exactly once. The decision was successfully obtained only in a few tens of nanoseconds. We argue that current fiber technology shall allow interrogating graphs of hundreds of nodes and providing a simple-to-implement alternative to a quantum computer approach.

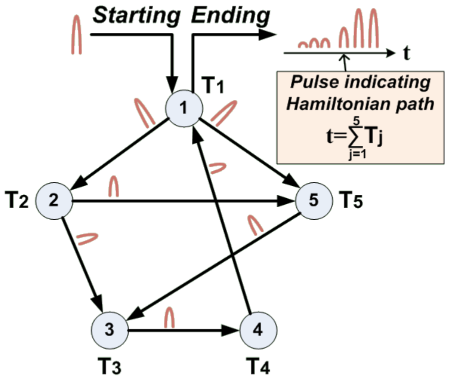

Fig.1 Optical network representation of a Hamiltonian path problem

We look at a graph consisted of 5 nodes (towns) connected by some directed paths (roads), as shown in Fig.1. Node 1 is set as both starting and ending node. It is easy to find that there exists a Hamiltonian path of 1(inject)→2→5→3→4→1. The idea is we inject an optical pulse into the graph from node 1 to mimick the behavior of a "travller". The pulse "traveller" will simultaneously try all the possible routes in the graph. For example, a pulse reaching node 2 from node 1 will be equally split to generate two pulses traveling from node 2 to node 3 and to node 5, respectively. We monitor the pulses returning to node 1. These pulses represent different routes in graph which all starts from and ends to node 1. These routes include three basic loops which are a) 1(inject)→2→3→4→1, b) 1(inject)→5→3→4 →1 and c) 1(inject)→2→5→3→4→1, and the conbinations of these basic loops such as 1(inject)→2→3→4→1→2→5→3→4→1. So what we need to do is to separate these pulses traveling along different routes and let them tell us whether a Hamiltonian path exists or not. The method we use is to assign delays to the nodes, i.e., node j has delay T_j ($j = 1\sim5$). The delay of each node is chosen such that its sum $\sum_{j=1}^{5} T_j$ can only be obtained by summing each node's delay exactly once. That means for a pulse visiting all the nodes exactly once, the delay it experienced is unique. If such a pulse from the returning pulses after a delay of $\sum_{j=1}^{5} T_j$ is observed, the Hamiltonian path exists., otherwise not exist. The experimental results are shown in Fig.2 (a). The delays of each node are 18.8 ns for node 1, 14.8 ns for node 2, 15 ns for node 3, 5 ns for node 4 and 28.4 ns for node 5. So the total delay $\sum_{j=1}^{5} T_j$ is 82 ns. The 3rd pulse has a delay of 82 ns, meaning that the Hamiltonian path exists. The 1st pulse travels along the route of 1(inject)→2→3→4→1 and the 2nd pulse travels along the route of 1(inject)→5→3→4→1. It can be seen that these pulses travelling along different routes have been separated succesfully. The 4th to 7th pulses travel multiple cycles in the graph. For example, the 4th pulse travels along 1(inject)→2→3→4→1→2→3→4 →1. Fig.2 (b) & (c) shows the output from node 3 and node 5 for

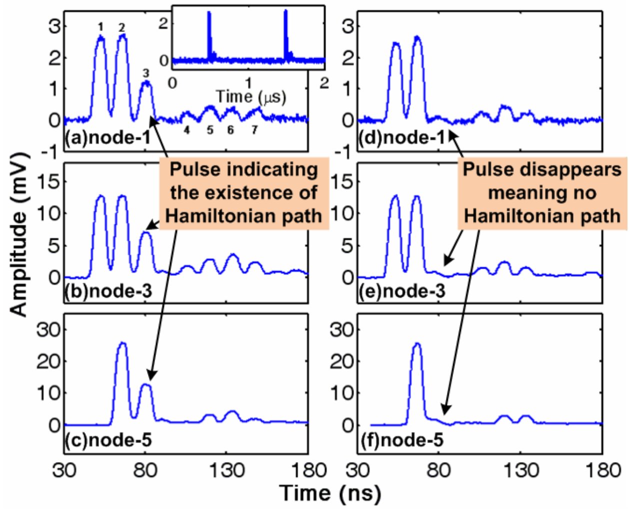

Fig.2 Pulse output from node 1, 3 and 5 with (a)-(c) and without (d)-(f) path 2→5 connected

monitoring purpose. If we break the connection 2→5, there is no Hamiltonian path any longer and it can be seen that the 3rd pulse disappears from the output of node 1, 3 and 5, as shown in Fig.2 (d)-(f), which verifies the validity of our optical method of determining the existence of Hamiltonian path.

In conclusion, we demonstrate an optical network approach to non-Turing optical computer to solve NP-complete problems. A proof-of-principle demonstration of solving directed Hamiltonian path problem was performed on a map with five towns built based on optical fiber network. The decision of the existence of Hamiltonian path can be made by monitoring the delayed output pulses from the fiber network where a positive answer means a pulse appearing at the delay equal to the total delay of the whole graph. The decision was successfully made in 82 ns. Considering the maximum obtainable pulse energy of ~10 µJ in fiber and minimum 10 photons for a reliable detection, we argue that current fiber technology shall allow interrogating graphs with up to a few hundred nodes. Moreover, it is known that all the NP-complete problems can be transferred to each other with a polynomial complete reduction, meaning that our optical computer can be applied to solve all kinds of NP-complete problems.