Market Crashes as Critical Points

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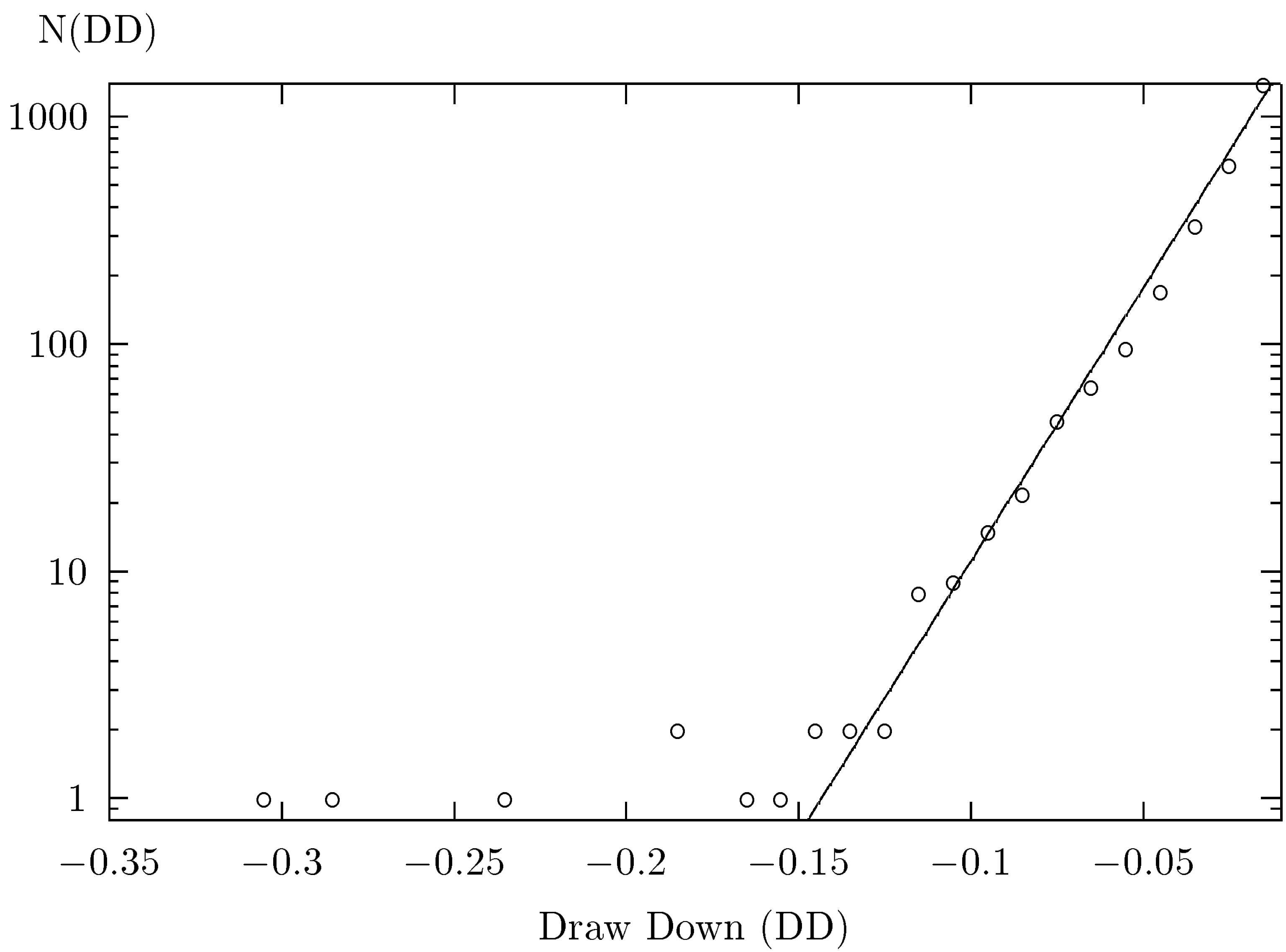
Stock Market Crashes

- In the last century, we can identify a total of five large market crashes:
  - 1914 (out-break of World War I),
  - October 1929 (triggering the Great Depressions of the 1930s),
  - October 1987 (the Black Monday),
  - July 1997 (onset of the Asian Economic Crisis), and most recently,
  - the NASDAQ crash of April 2000.

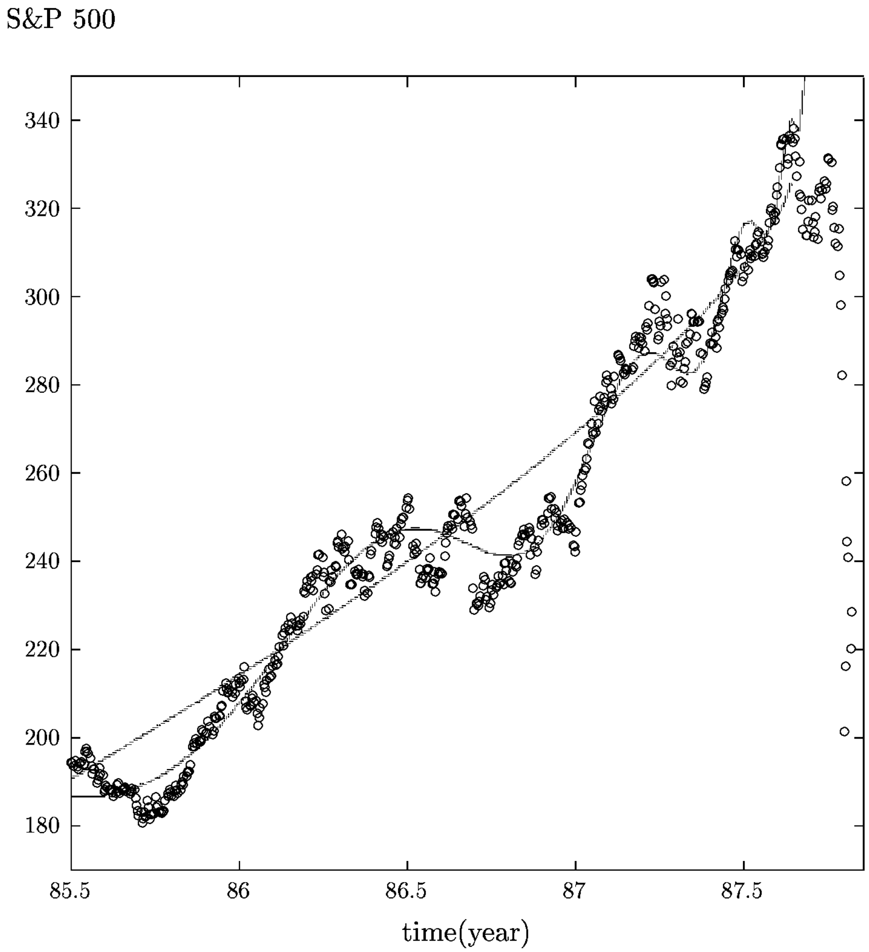
Large Market Crashes are Extraordinary!

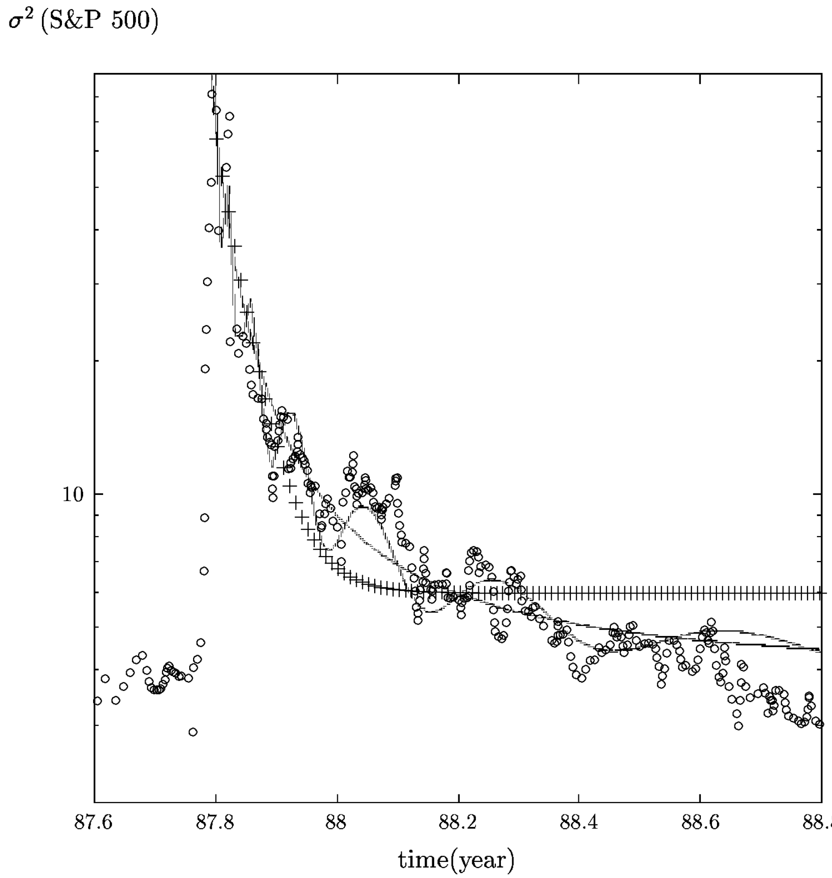
- By plotting the cumulative frequencies of daily loses in the stock market against the log-magnitude of the loss, Sornette *et al* found that:
  - "normal day-to-day" trading results in an exponential daily loss distribution.
  - the above large crashes are statistical outliers, very much out of the ordinary.


N(DD)
1000
100
10
1
−0.35
−0.3
−0.25
−0.2
−0.15
−0.1
−0.05
Draw Down (DD)

Market Crashes and Earthquakes

Precursors and Aftershocks





Modeling Market Crashes — Macroscopic Considerations

- Rational market with incomplete information: not every aspect of the market is known, but whatever is known is reflected in the price of stocks and their fluctuations.

- Price of stock reflects not only the fundamental worth of whatever it represents, but also possible future gains, which comes at a risk (or hazard rate).

- The more risky the stock, the higher it is priced.

Modeling Market Crashes — the Microscopic Model of Sornette and Johansen

- System of N traders in a trading network, in which each trader $i = 1, \ldots, N$ is connected to $N(i)$ nearest neighbors in its neighborhood $\mathfrak{N}_i$ according to some graph.

- Traders interact only locally via such a network.

- In this highly simplified model, each trader i can only be in one of two states $s_i = -1$ (BUY) or $s_i = +1$ (SELL) at any one time, reflecting the major pre-occupation of the trader at that time.

- Decision process of trader i only influenced by
  - opinions of the $N(i)$ traders in its neighborhood;
  - an idiosyncratic signal received by trader i alone.
- Time evolution governed by a cellular automaton rule:

$$s_i(t+1) = \text{sign}\left[K \sum_{j \in \mathfrak{N}_i} s_j(t) + \sigma \epsilon_i(t)\right], \qquad i = 1, \ldots, N.$$

- K = coupling strength — orders the system of traders, σ = strength of noise term — disorders the system of traders. Relative size of K and σ determines whether system is ordered or disordered.
- In this language, a market crash occurs whenever instantaneous correlations due to local fluctuations get magnified to $O(N)$ proportions by the positive feedback intrinsic in the interactions.

Results from the Sornette-Johansen Model

- Existence of critical points K_C on most networks.

- Susceptibility χ = sensitivity of system to small perturbations diverges as power law as K approaches K_C from below:

$$\chi \sim A(K_C - K)^{-\gamma},$$

where $\gamma > 0$ is the *critical exponent* of the susceptibility.

- Assume that the system is driven exogeneously slowly, such that K approaches K_C linearly as

$$K_C - K(t) \approx \alpha(t_C - t),$$

where the critical time $t_C =$ most probable time for market crash.

- Reasonable to assume that the average hazard rate $h(t)$ in the market should be positively correlated with $\chi(K(t))$, i.e.

$$h(t) \sim B(t_C - t)^{-\alpha},$$

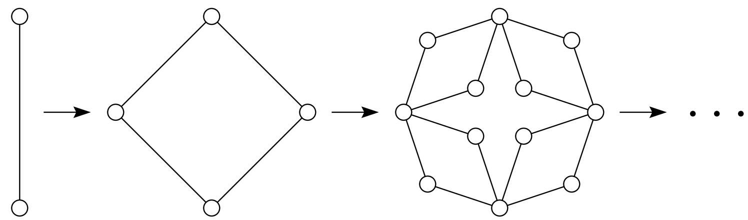
  for $0 < \alpha < 1$ (so that the stock price remains finite at the critical point).

- Rational Expectation Model implies

$$p(t) = \int_{t_0}^{t} h(t')\, dt',$$

  where $p(t) =$ price of stock, or stock index at time $t \implies$ power-law acceleration of price increase near t_C.

Additional Results of Model on Hierarchical Lattice



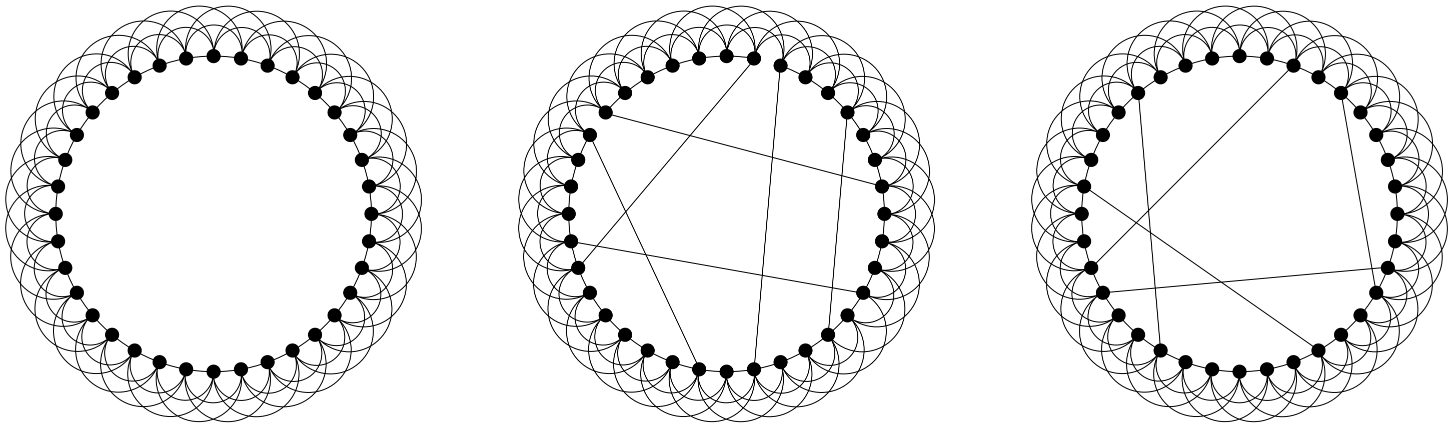
- Power-law divergence of χ decorated by *log-periodic oscillations* due to discrete scale invariance in hierarchical lattice;

$$\chi \sim A_0'(K_C - K)^{-\gamma} + A_1'(K_C - K)^{-\gamma} \cos\left[\omega \log(K_C - K) + \psi\right] + \cdots \ .$$

- Such log-periodic oscillations reflected in $p(t)$ too;

- Sornette *et al* and Feigenbaum *et al* fitted market data to extract t_C — reasonable agreement. (include Feigenbaum's graphs)

Examples of Small-World Networks



(a) (b) (c)

(a) = regular 1-dimensional clustered lattice with range of interaction k,
(b) = Watts-Strogatz small-world network — randomly rewiring qkN bonds,
(c) = Newman-Watts small-world network — addition of qkN random bonds.

Majority-Rule

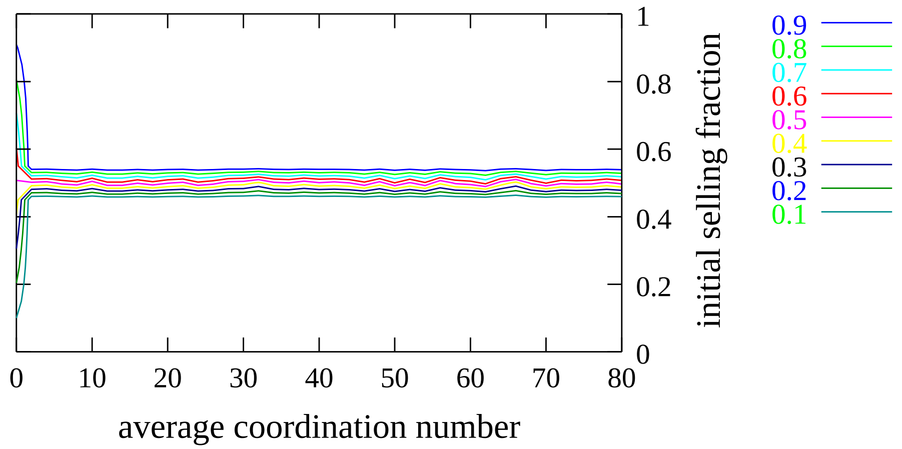
- I chose states $s_i \in \{0, 1\}$ to use boolean variables. $0 =$ BUY, $1 =$ SELL.

- Majority-rule to generate time evolution. Stochastic parameter $p =$ probability that trader will take risk to change trading strategy when local trading network ambivalent.

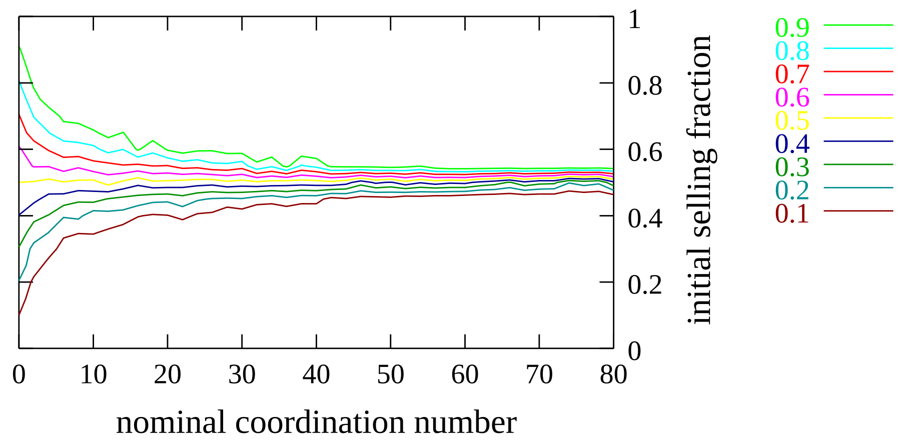
$$s_i(t+1) = \begin{cases} \text{MAJORITY}\left[s_i(t); \{s_j(t) \mid j \in \mathfrak{N}_i\}\right], & \text{if } R_i(t) < 1 - p, \\ \text{MAJORITY}\left[\text{NOT}[s_i(t)]; \{s_j(t) \mid j \in \mathfrak{N}_i\right], & \text{otherwise;} \end{cases}$$

where MAJORITY-function returns 0 if majority of traders buying and 1 if majority of traders selling. NOT is binary negation.

Preliminary Results



Random Network

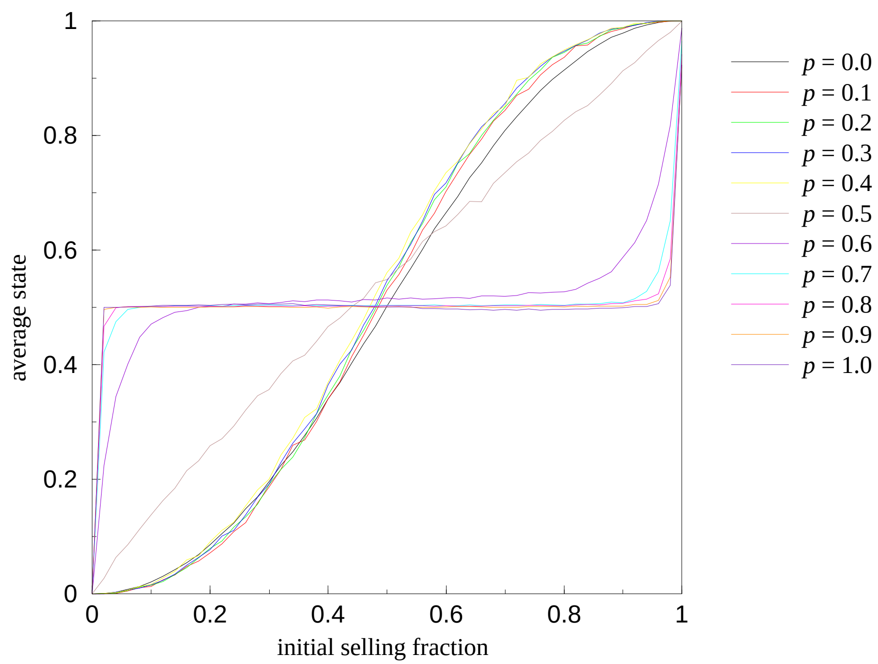


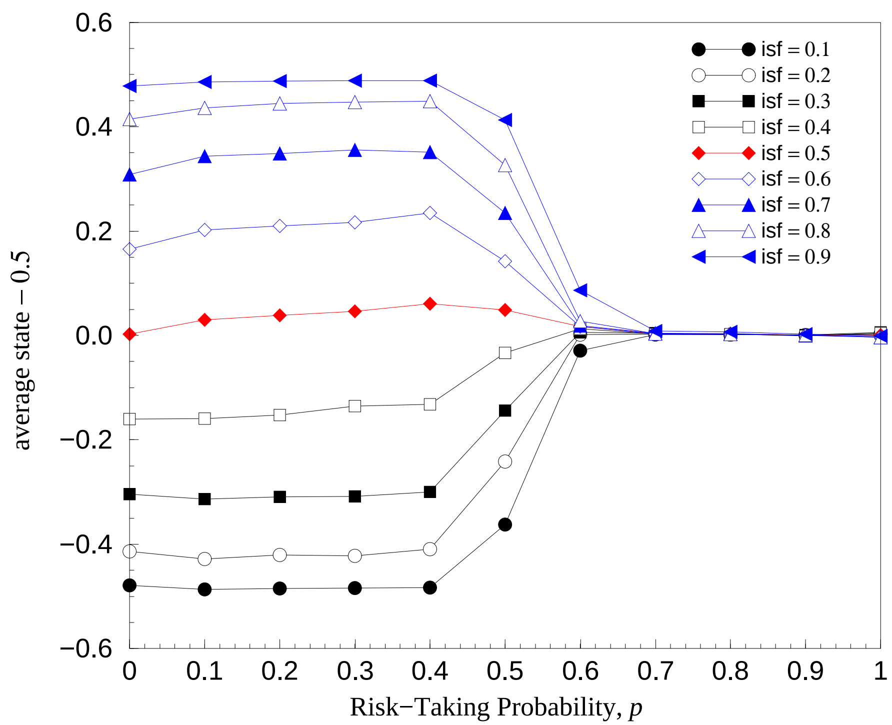
Newman-Watts Small-World Network

- Equilibrium state when roughly half of the traders buying and half of the traders selling;

- Equilibrium state of random network much more sensitive to small perturbations than that for Newman-Watts small-world network.

Continuous Phase Transition for $k = 1$!





Asymptotic average state $\langle s_i(\infty) \rangle$ for various p

Order parameter $m(p) = \langle s_i(\infty) \rangle - \frac{1}{2}$ as a function of p

- Signs of continuous phase transition;

- $k = 1$ corresponds to 1-dimensional Ising model. If $p \leftrightarrow T$, then $T_C = 0$ for 1-dimensional Ising model $\Longrightarrow$ $p_C = 0$. But not the case: $p_C > 0$!

- No such qualitative changes for $k > 2$.

Further Investigations

- Critical exponent of $k = 1$ phase transition in Newman-Watts small-world network;

- Comparision between majority-rule and Sornette-Johansen sign-rule — can we have phase transitions for $k > 2$ Newman-Watts small-world networks?

- Sornette and Johansen used hierarchical lattices — real world traders organized into hierarchies. But hierarchical lattice exhibit no clustering — Newman-Watts random rewiring to give hybrid hierarchical small-world networks?

- Stock index as an endogeneous global influence term?

- Fundamental diagram for the stock market?

- Random update rules and effective update rules?

Comments, suggestions and collaborations welcomed!