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Time Series Approaches to Understanding Earthquake Dynamics

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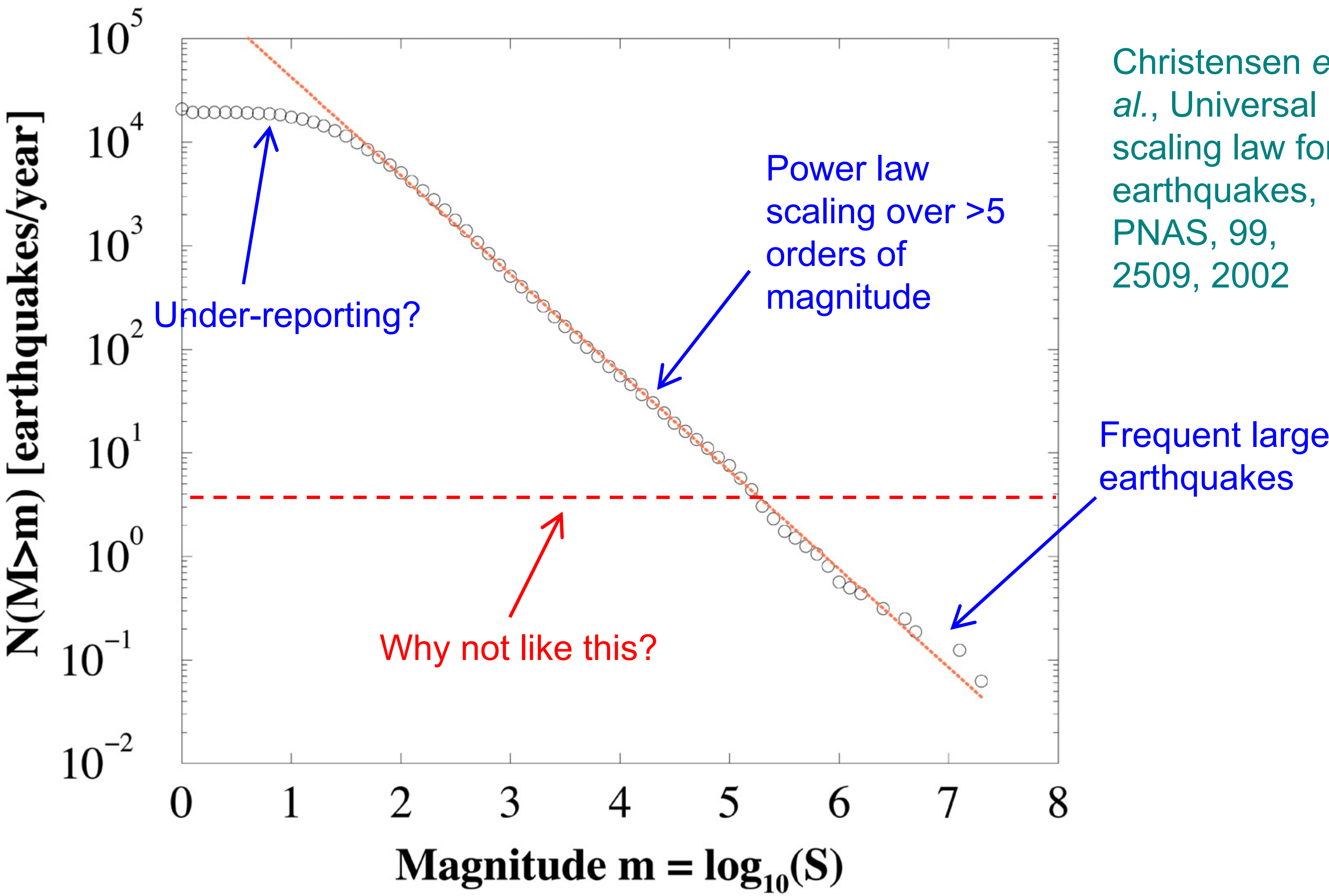
Gutenberg-Richter Law

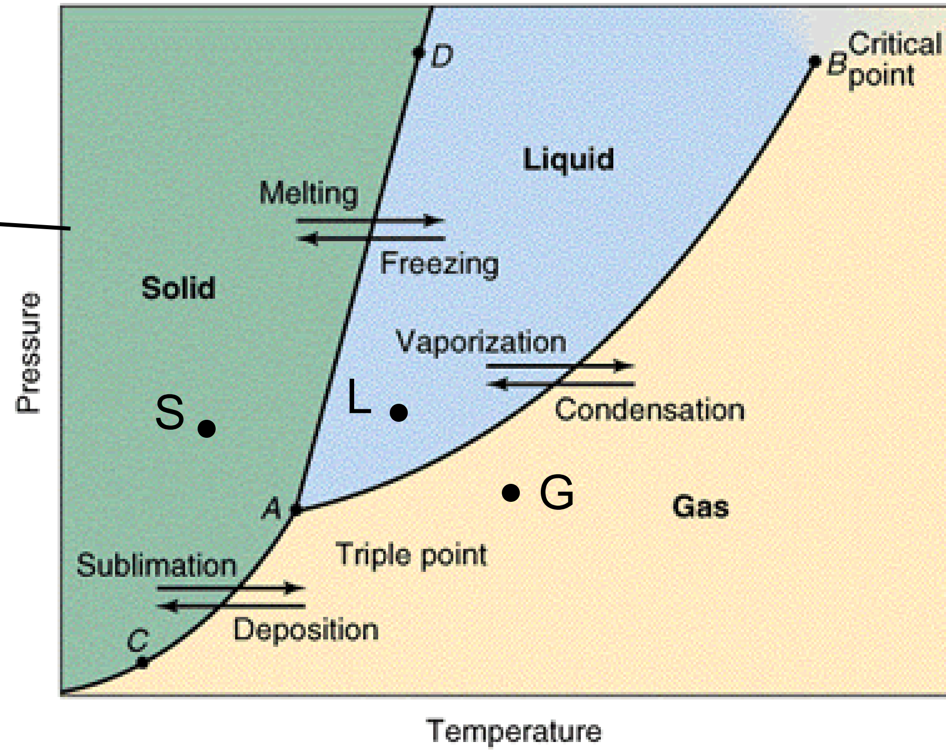


Christensen *et al.*, Universal scaling law for earthquakes, PNAS, 99, 2509, 2002

Self Organization

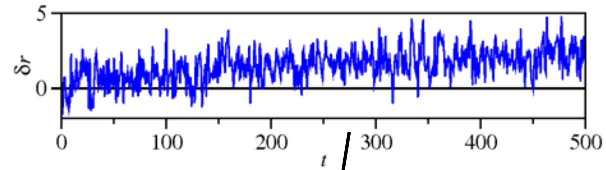
- Complex earth crust dynamics
  - Competing processes
  - Information/energy flow between different time/length scales
- Transitions between dynamical phases
  - Earthquakes
- Statistical Signatures
  - Critical slowing down
    - Dakos *et al*., PNAS **105**, 14308 (2008)
    - Scheffer *et al*., Nature **461**, 53 (2009)
  - Synchronization
    - Leung, Phys Rev E **58**, 5704 (1998)
    - Osipov *et al*., Phys Rev Lett **91**, 024101 (2003)
    - Goo *et al*., arXiv:0903.2099, 12 Mar 2009

Macroscopic Physics

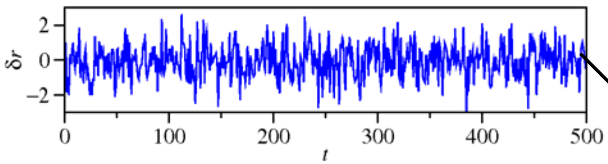


- Macroscopic order parameters differentiate
  - Solid (S)
  - Liquid (L)
  - Gas (G)

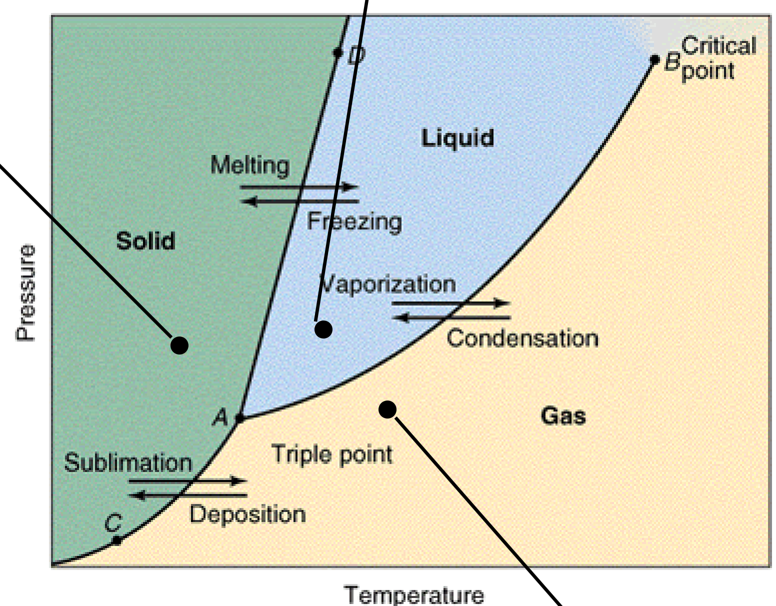
Microscopic Physics



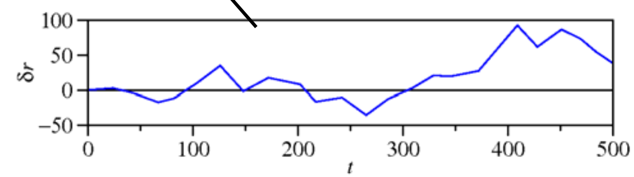
diffusive trajectories,
δr^2 increases with time



δr fluctuates about 0,
$\delta r^2 = \alpha T$ time-independent

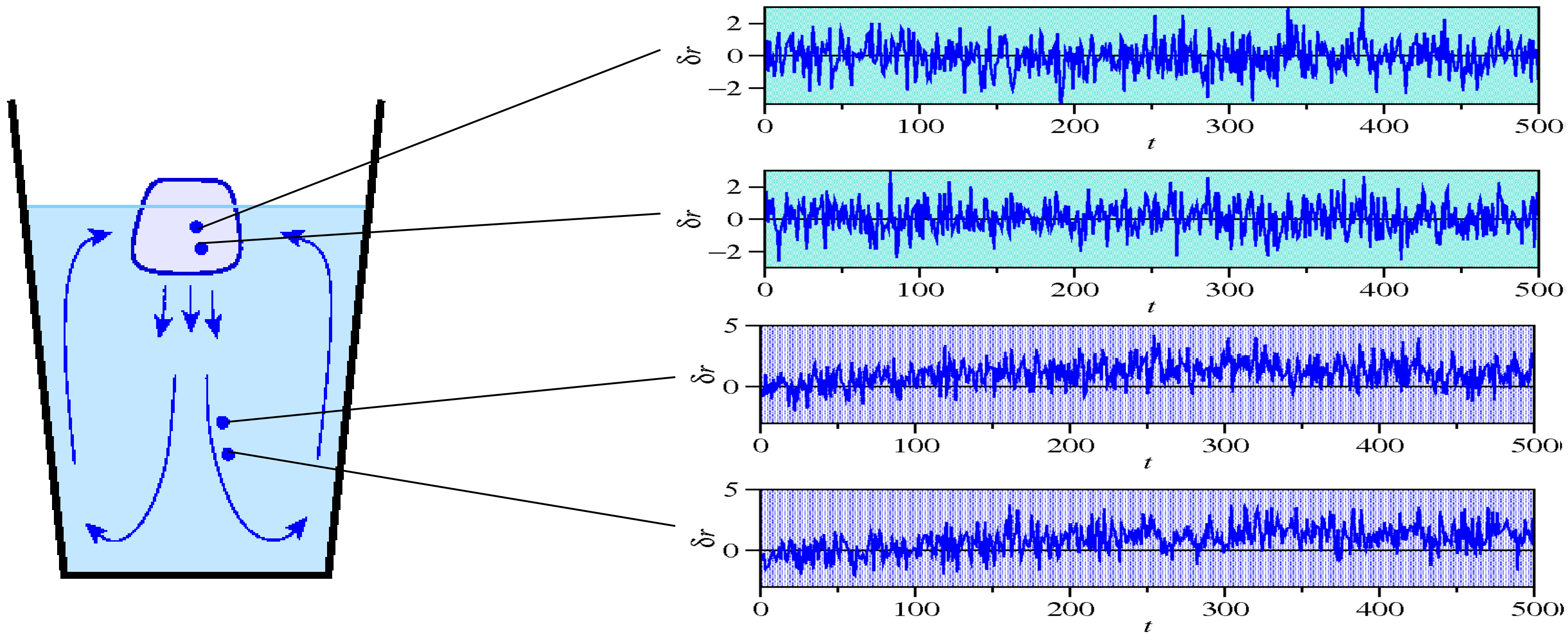


ballistic trajectories,
infrequent collisions



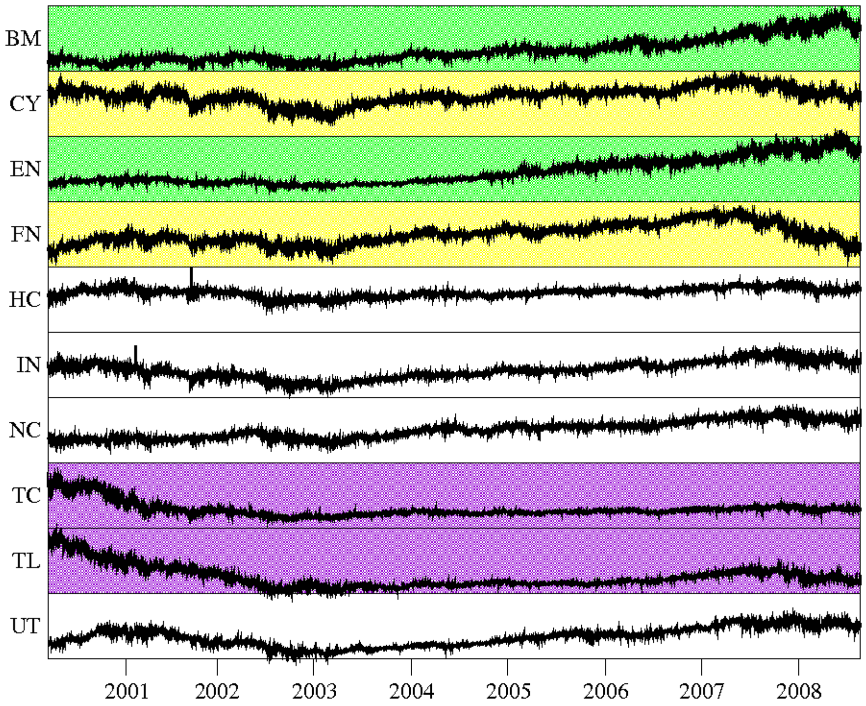
- S, L, G time series distinguishable
- S, L, G phase within single time series distinguishable

Macro ↔ Micro



- Group statistically similar time series
- Discover presence of different phases

Cross Correlations Between Time Series



Dow Jones US economic sector indices

$$C_{ij} = \frac{\left\langle (x_i - \bar{x}_i)(x_j - \bar{x}_j) \right\rangle}{\sigma_i \sigma_j} = \left\langle \frac{\delta x_i}{\sigma_i} \frac{\delta x_j}{\sigma_j} \right\rangle$$

↓

$$D_{ij} = \left\langle \theta\left(\frac{\delta x_i}{\sigma_i} - 1 \right) \theta\left(\frac{\delta x_j}{\sigma_j} - 1 \right) \right\rangle$$

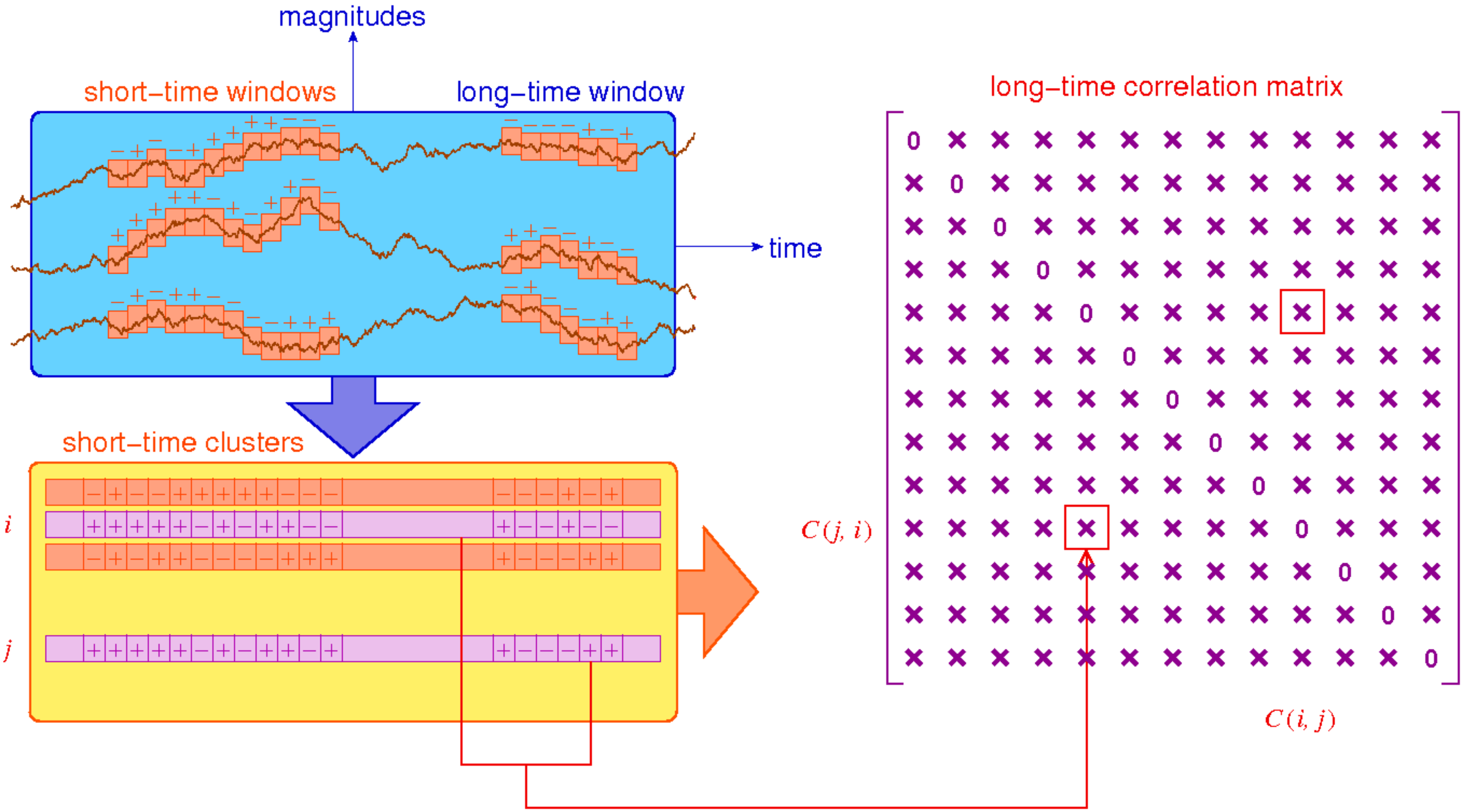
digital cross correlations

↓

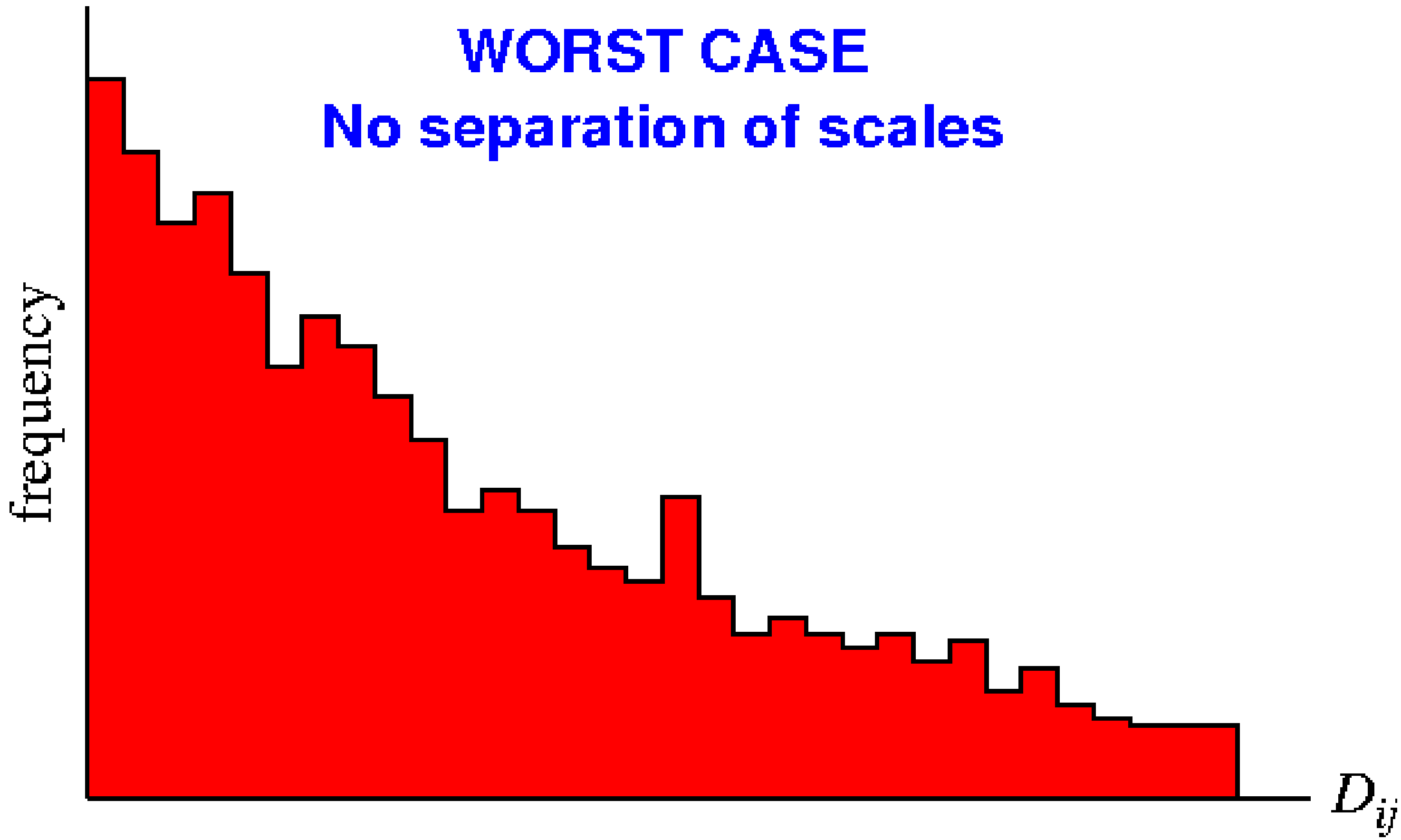
$$\widetilde{D}_{ij} = \left\langle \theta(\Delta x_i \Delta x_j) \right\rangle \text{ or } \widetilde{D}_{ij} = \sum_{t=1}^{N} \theta(\Delta x_{it} \Delta x_{jt})$$

comovement digital cross correlations

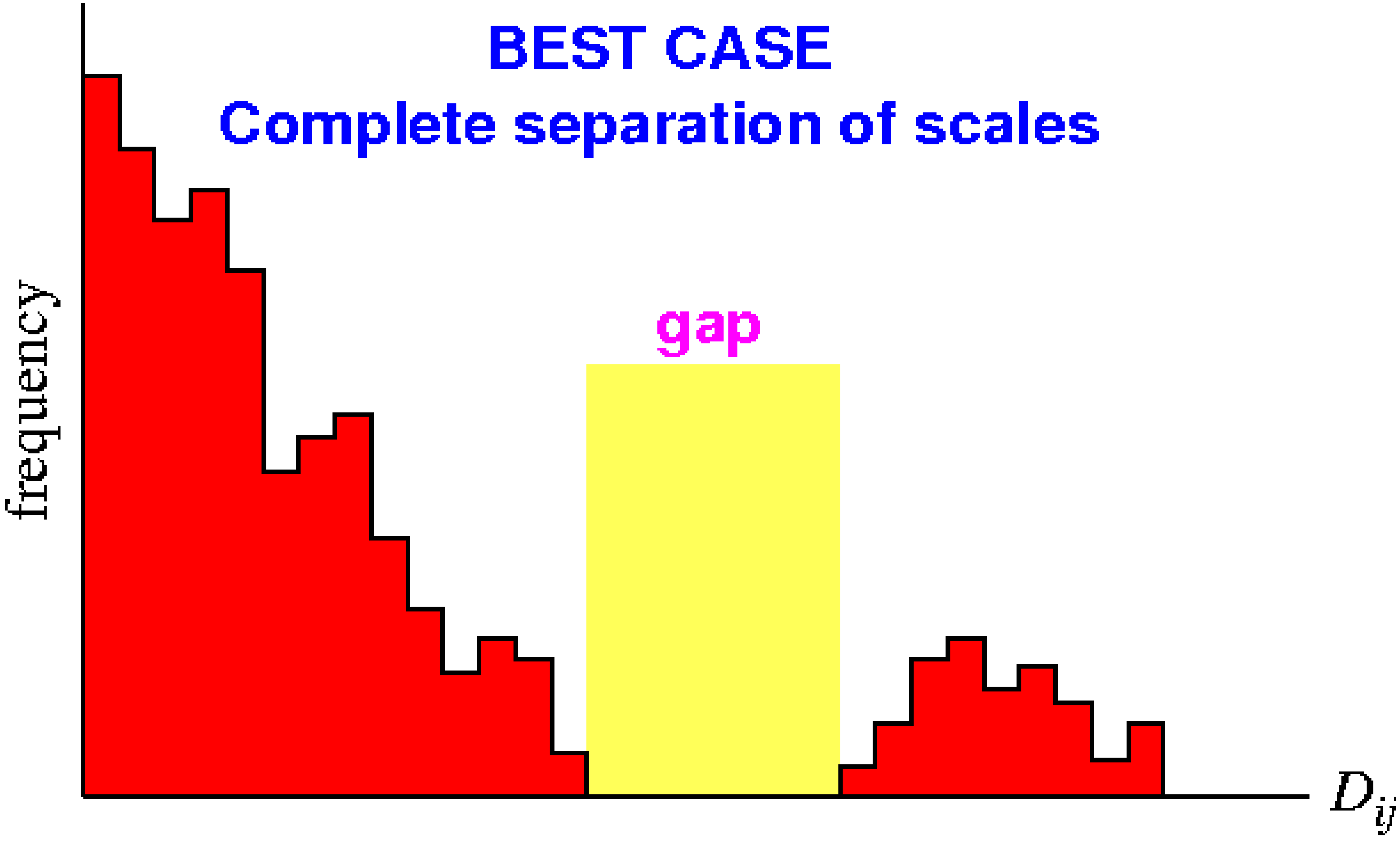
Long-Time Correlation Matrix



No Separation of Scales

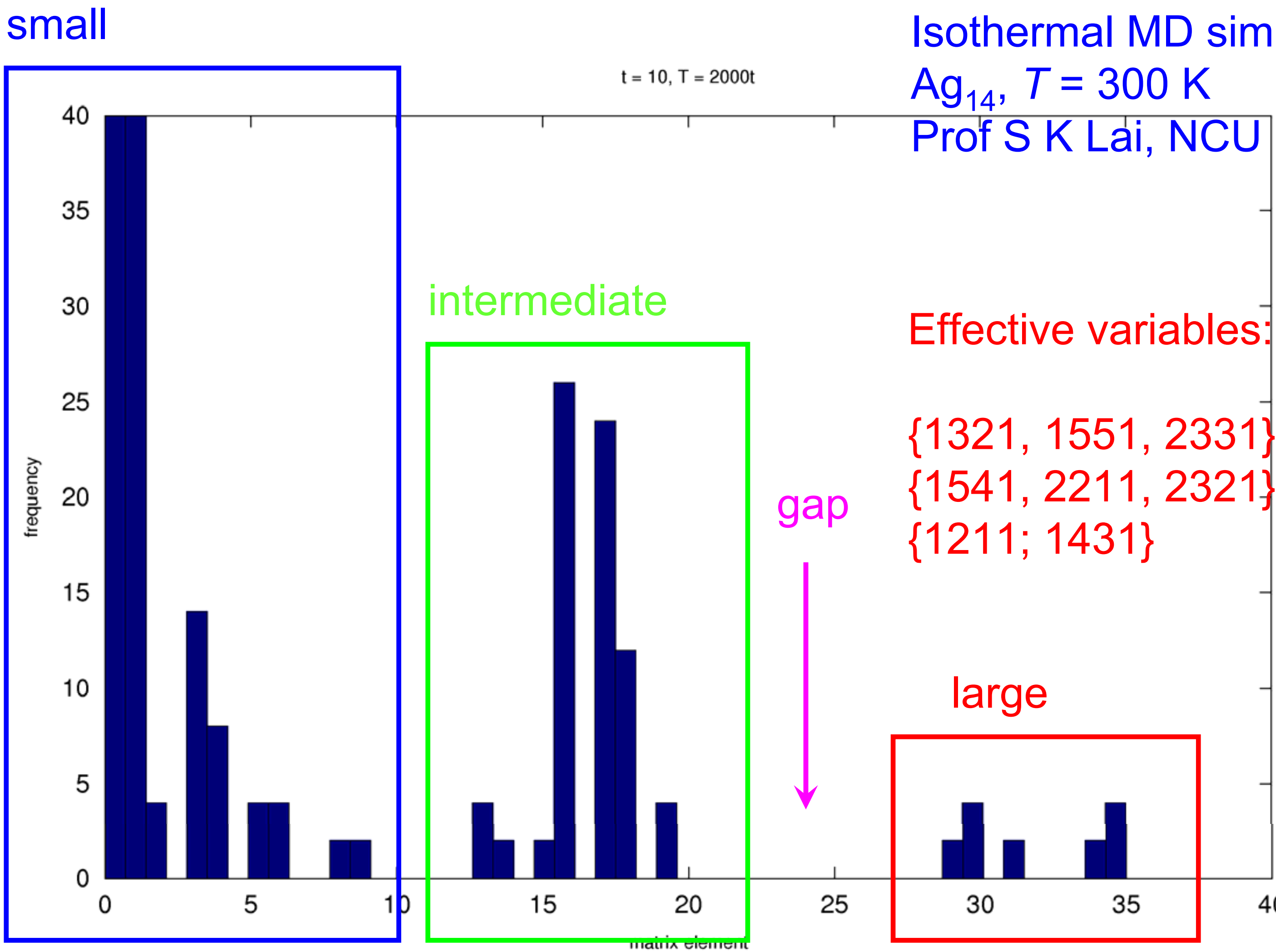


Complete Separation of Scales



Metallic Nanoclusters

Isothermal MD simulations
Ag_{14}, T = 300 K
Prof S K Lai, NCU Physics



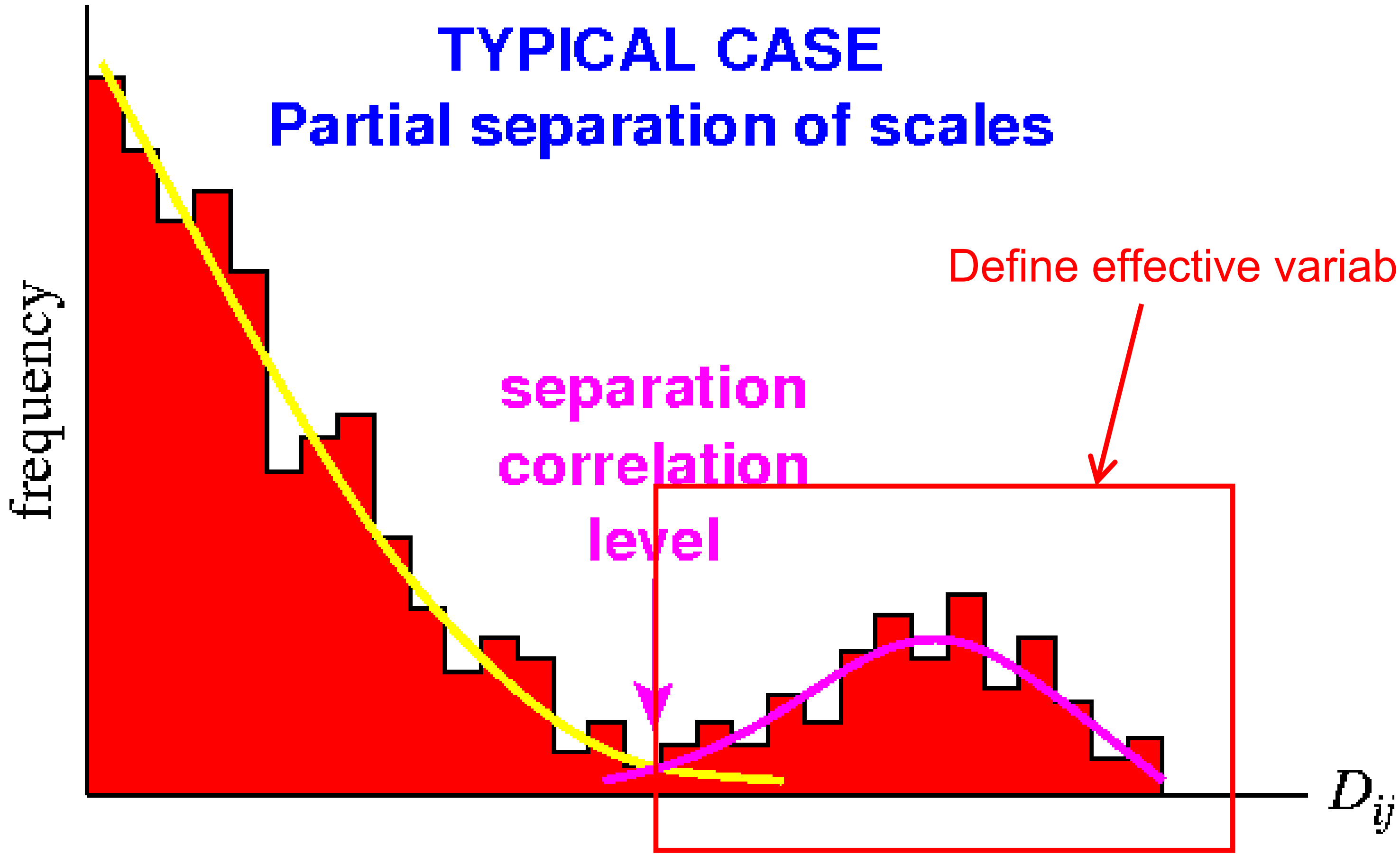
Effective variables:

{1321, 1551, 2331}
{1541, 2211, 2321}
{1211; 1431}

Discover Effective Variables

	1321	1551	2331	1541	2211	2321	1211	1431
1321	0	35	35	1	3	1	17	18
1551	35	0	34	0	3	1	16	17
2331	35	34	0	1	2	0	17	17
1541	1	0	1	0	30	29	15	18
2211	3	3	2	30	0	30	13	16
2321	1	1	0	29	30	0	13	14
1211	17	16	17	15	13	13	0	31
1431	18	17	17	18	16	14	31	0

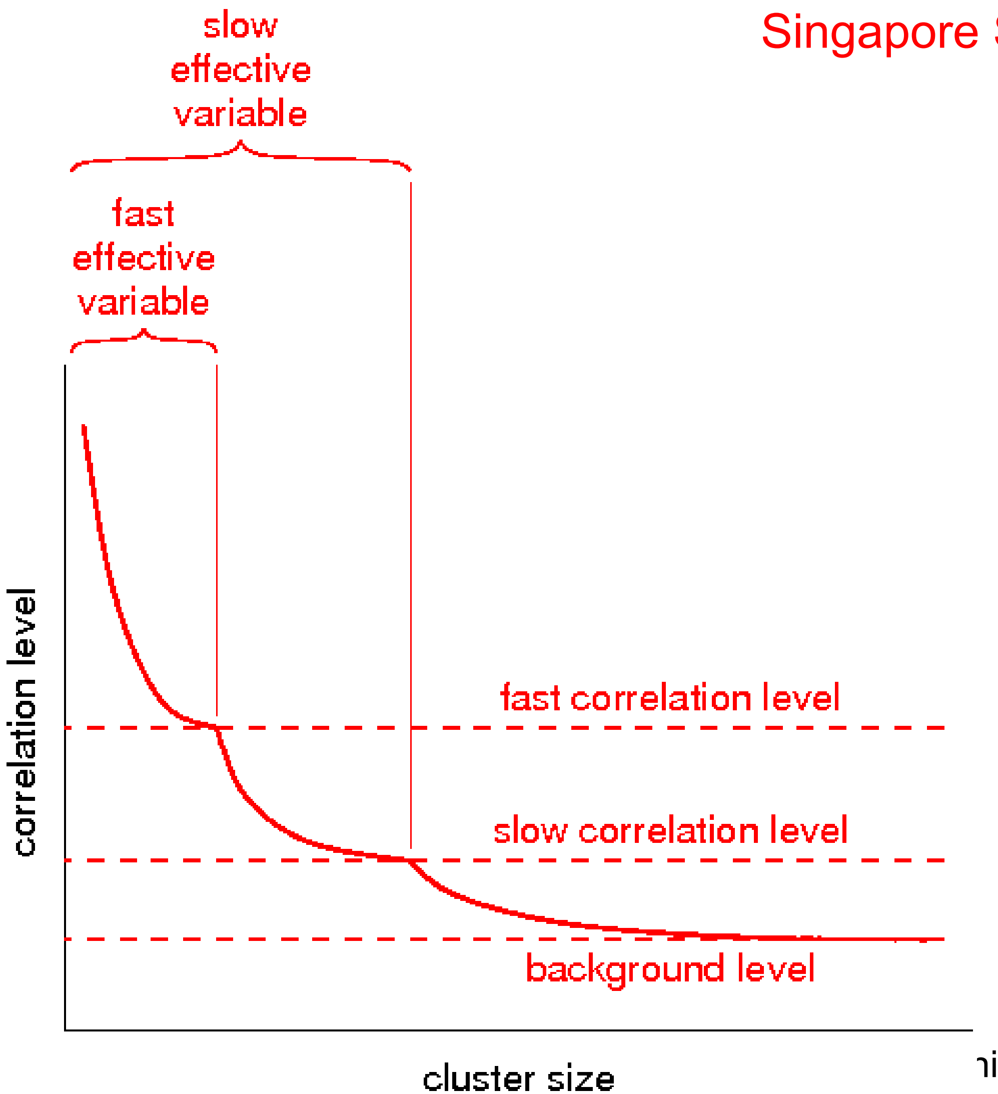
Partial Separation of Scales



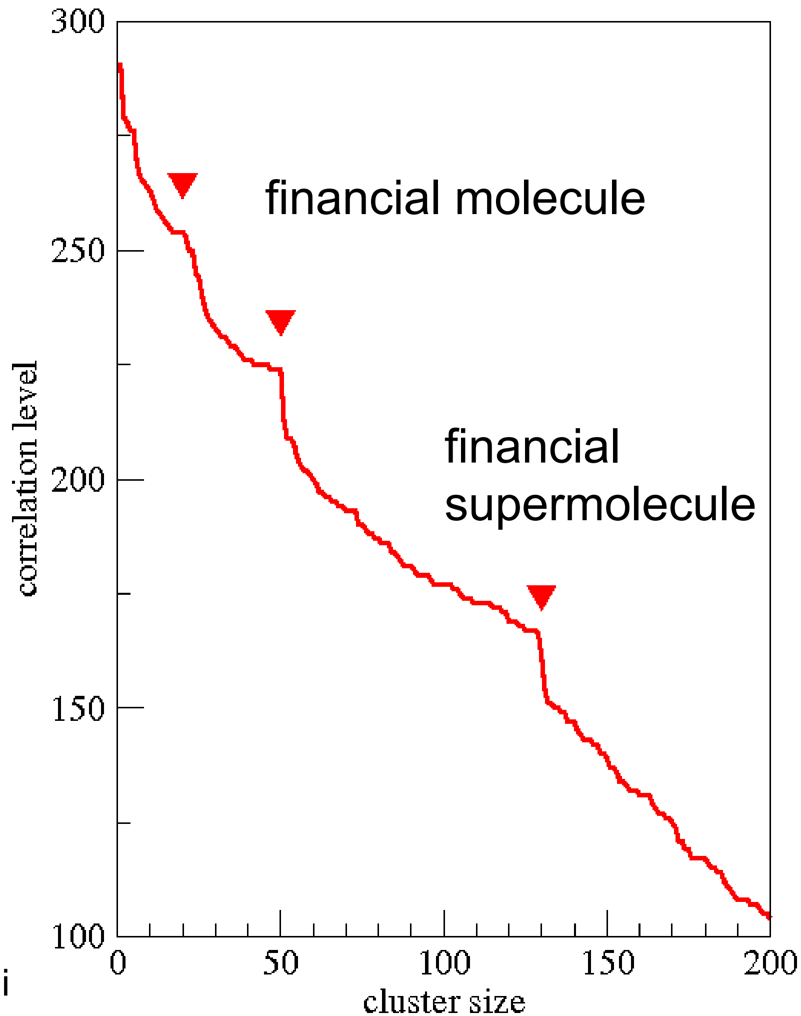
Partial Hierarchical Clustering

- Seed cluster
  - Find $D_{i^*j^*} = \max_{ij} D_{ij}$
  - Use $c = \{i^*, j^*\}$ as seed cluster
- Grow cluster
  - Add k^* to cluster if
    - $D_{k,c} = \min_{l \in c} D_{kl}$
    - $D_{k^*,c} = \max_k D_{k,c}$
  - Iterate
- Cluster boundary
  - Plot correlation level $D_{k^*,c}$ against cluster size

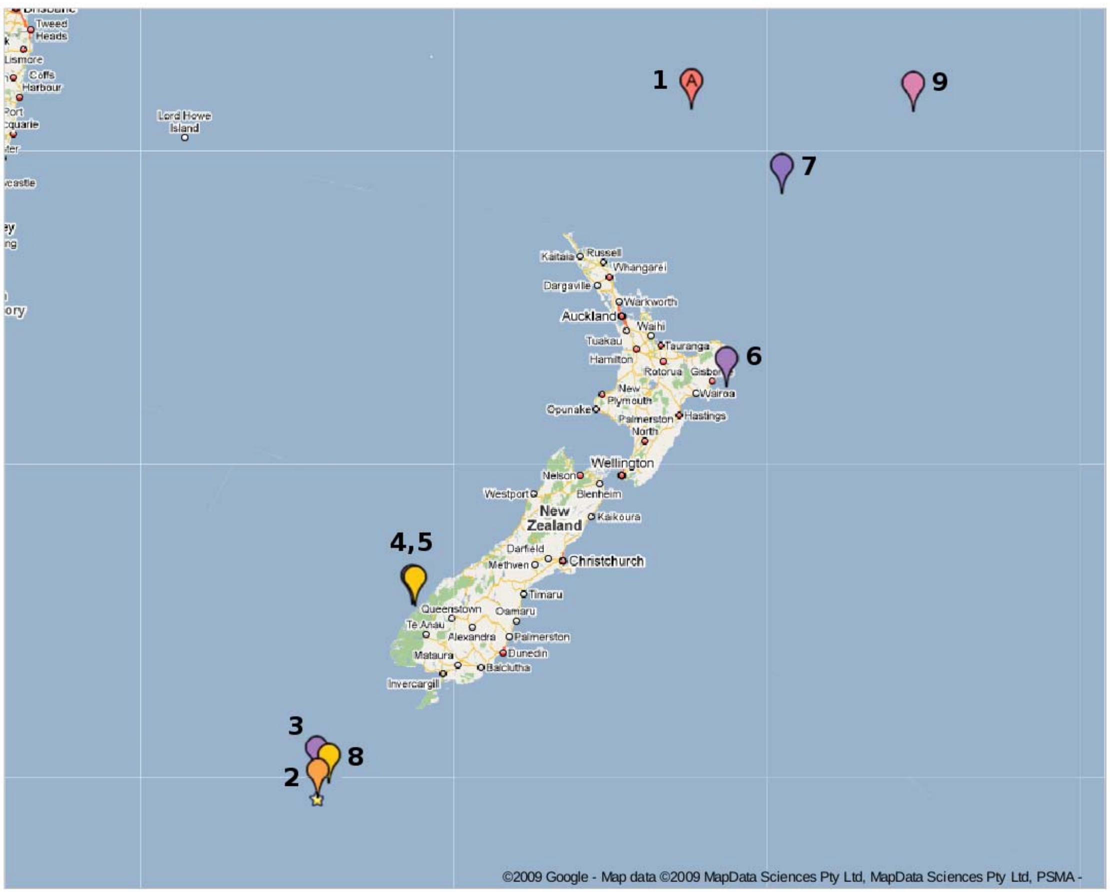
Hierarchy of Effective Variables



Singapore Stock Exchange (SGX): 2006-2007

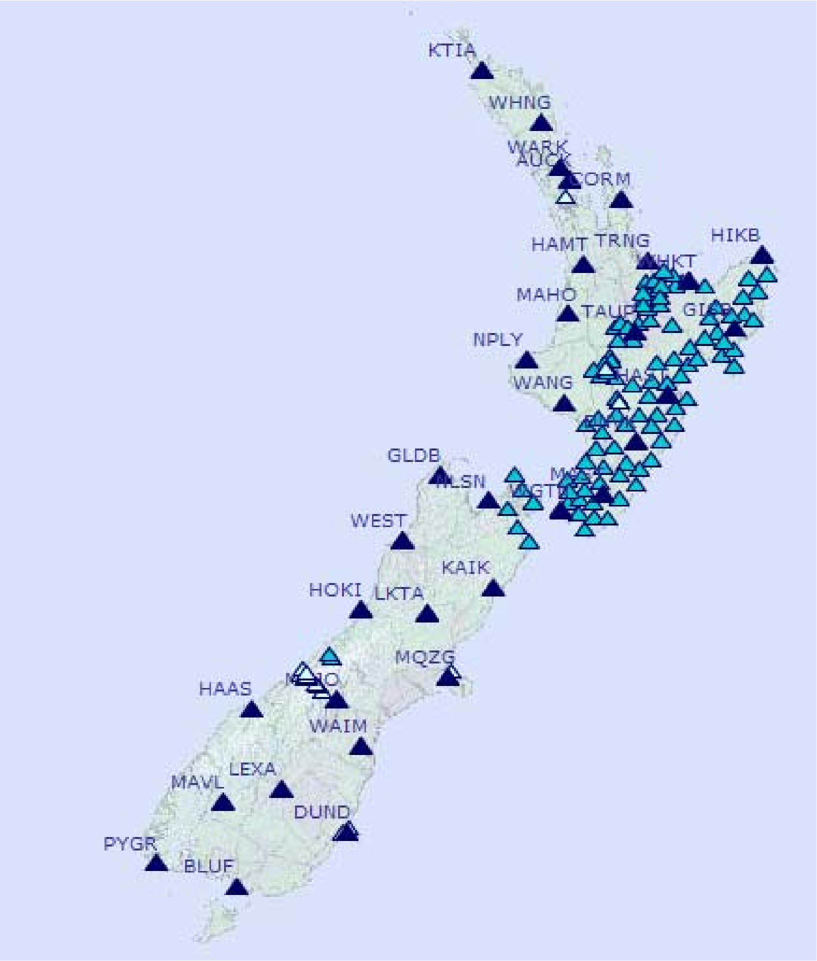


M > 6 New Zealand Earthquakes



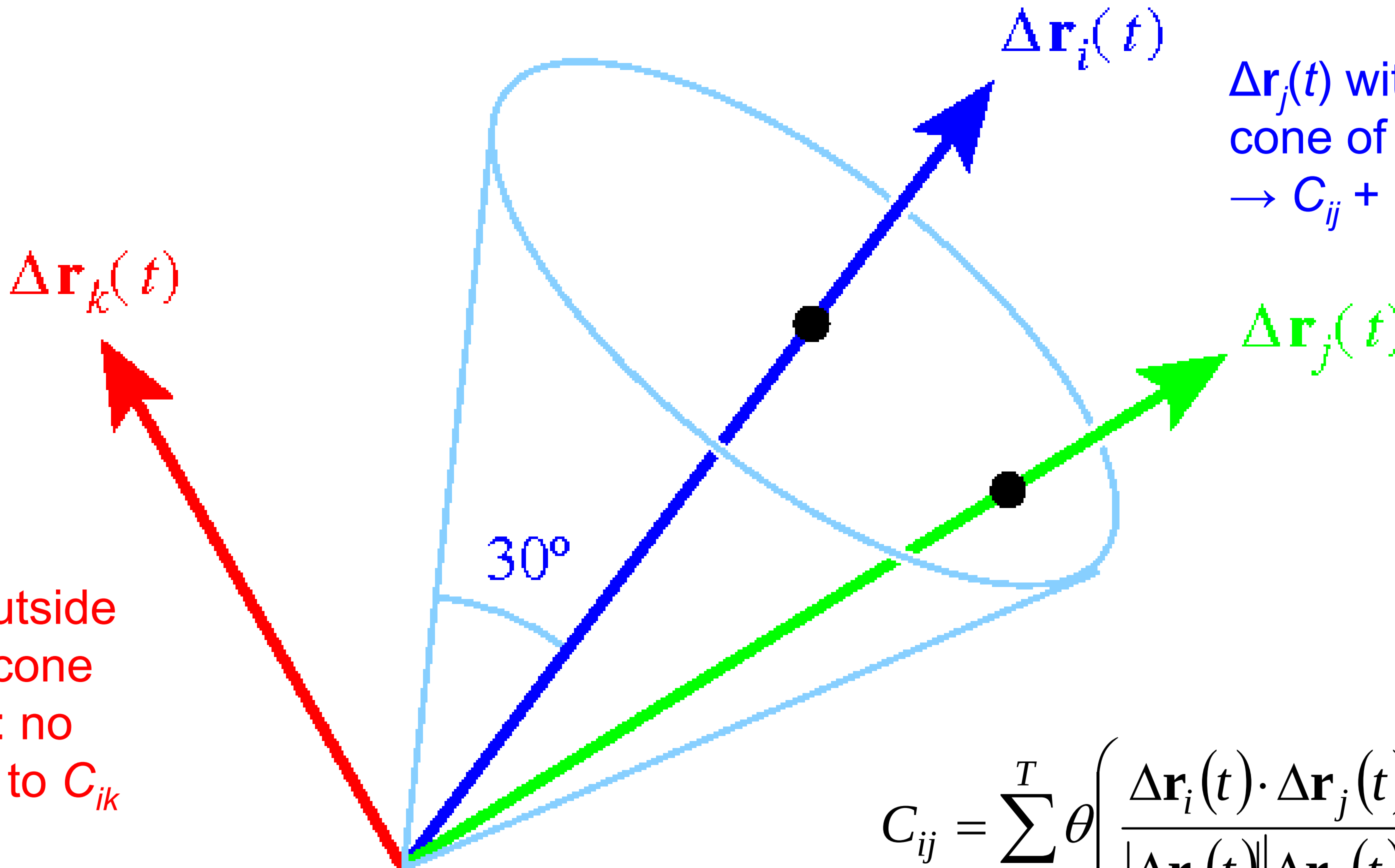
S/No.	Universal date & time
Spatial Cluster 1	
1	26 Dec 2006 at 21:06
6	20 Dec 2007 at 07:55
7	28 Mar 2008 at 06:39
9	29 Sep 2008 at 15:19
Spatial Cluster 2	
2	30 Sep 2007 at 05:23
3	30 Sep 2007 at 09:47
8	26 Apr 2008 at 23:34
Spatial Cluster 3	
4	15 Oct 2007 at 12:29
5	15 Oct 2007 at 21:28

The New Zealand GPS Network



- 100+ stations
- Oct 2006 to Mar 2008
- Daily time series data
  - $\mathbf{r} = (x, y, z)$
  - $\Delta\mathbf{r} = (\Delta x, \Delta y, \Delta z)$

Digital Vector Cross Correlations



$\Delta \mathbf{r}_j(t)$ within 30° cone of $\Delta \mathbf{r}_i(t)$: $C_{ij} \rightarrow C_{ij} + 1$

$\Delta \mathbf{r}_k(t)$ outside of 30° cone of $\Delta \mathbf{r}_i(t)$: no change to C_{ik}

$$C_{ij} = \sum_{t=1}^{T} \theta\left(\frac{\Delta \mathbf{r}_i(t) \cdot \Delta \mathbf{r}_j(t)}{|\Delta \mathbf{r}_i(t)||\Delta \mathbf{r}_j(t)|} - \frac{\sqrt{3}}{2} \right)$$

Synchronized Clusters

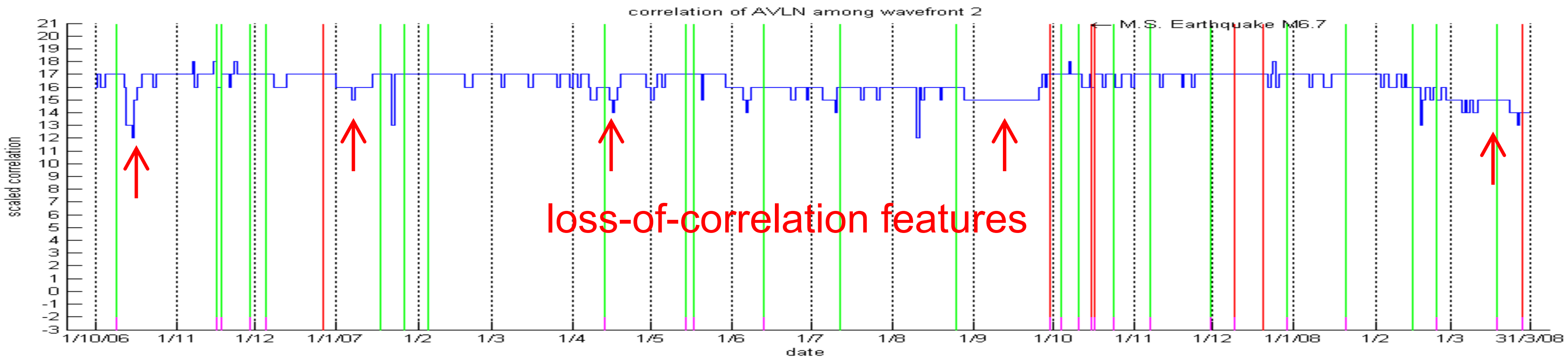
Synchronized clusters	***Stations***	***Geographical location***
SC1	HAMT, TRNG, AUCK, CORM, MAHO, WHNG, NPLY, RIPA, MARW	North Island
SC2	AVLN, KAPT, WGTT, PARW, TINT, KAIK, WAIM, MAST, TRAV, CHAT, LEXA, CLIM, MQZG, BLUF, DUND, MTJQ, OUSD, DUNT, LYTT, CMBL	Southern coast, South Island
SC3	QUAR, WEST, GLDB, WANG, HOKI, VGPK, VGMT, GISB	Northern coast, South Island

Internal Dynamics of Synchronized Clusters

$$C_{ij} = \sum_{t=1}^{T} \theta\left(\frac{\Delta\mathbf{r}_i(t)\cdot\Delta\mathbf{r}_j(t)}{|\Delta\mathbf{r}_i(t)||\Delta\mathbf{r}_j(t)|} - \frac{\sqrt{3}}{2} \right) \longrightarrow g_i(t) = \sum_{\substack{j\neq i;\\ j\in SC}} \theta\left(\frac{\Delta\mathbf{r}_i(t)\cdot\Delta\mathbf{r}_j(t)}{|\Delta\mathbf{r}_i(t)||\Delta\mathbf{r}_j(t)|} - \frac{\sqrt{3}}{2} \right)$$

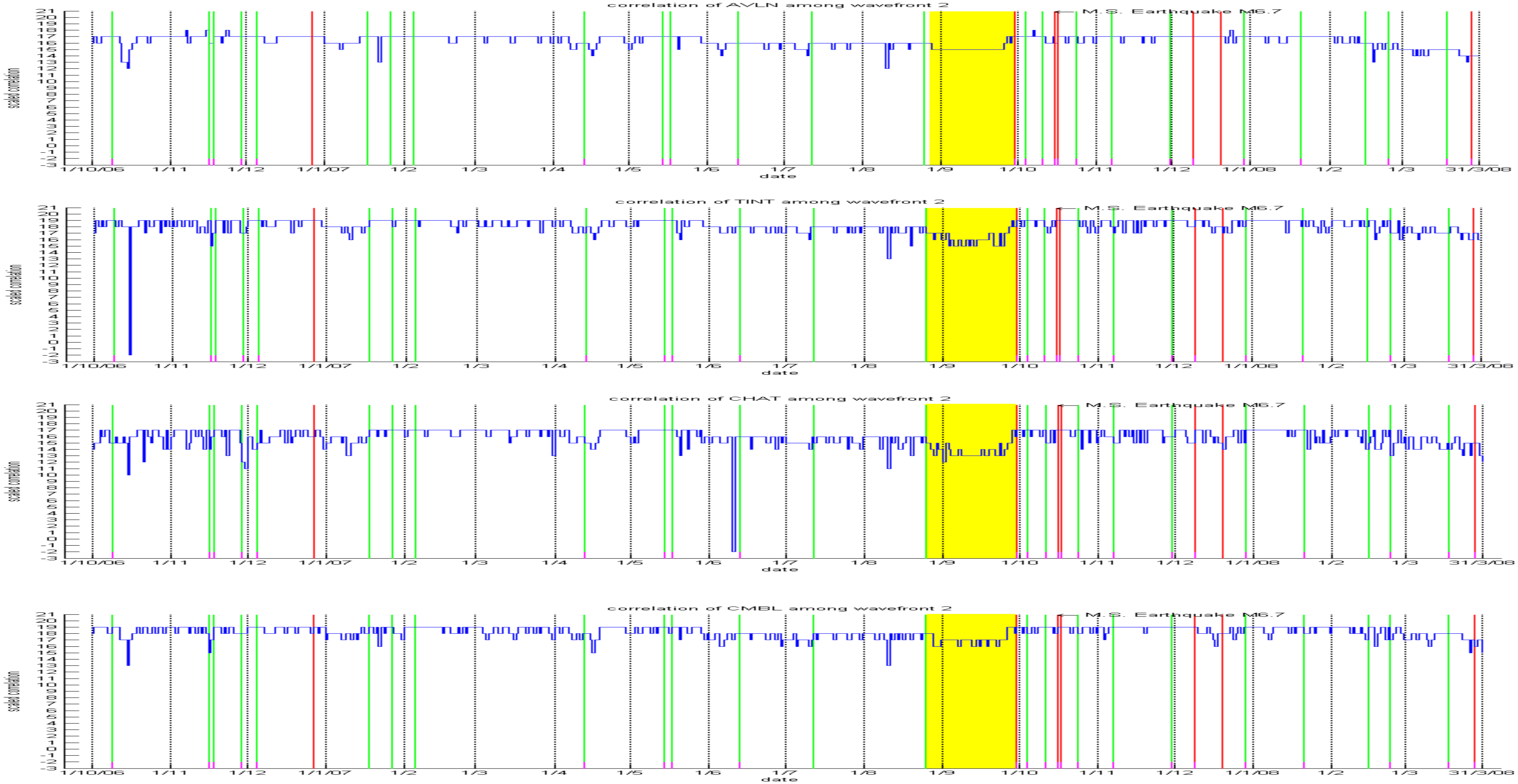
Longitudinal
Discover synchronized clusters

Cross-sectional
Across synchronized clusters

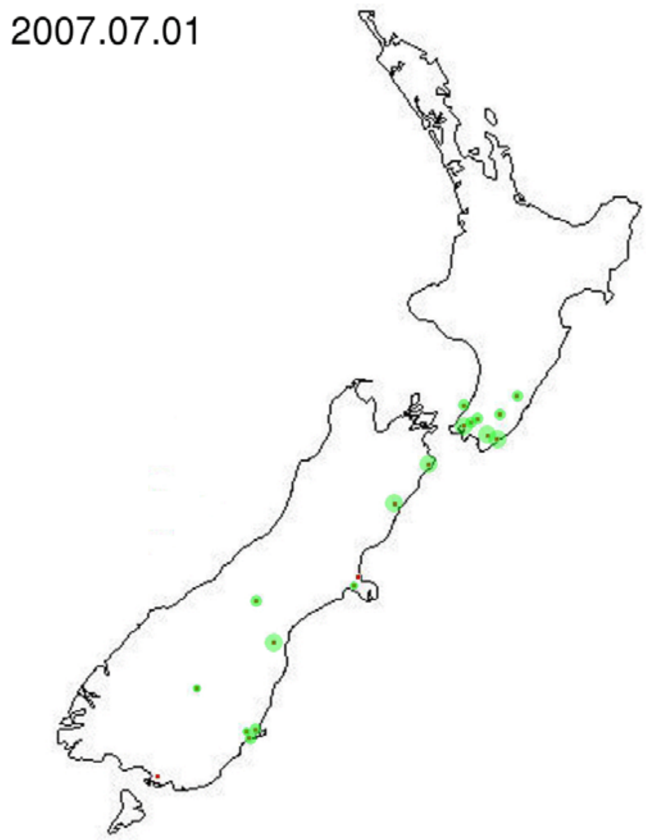


Precursor Feature for SC2

$5 < M < 6$ **precursor feature** **$M > 6$**



Spatio-Temporal Dynamics of Precursor Feature



Segmentation vs Clustering

- Time Series Clustering
  - Discover effective mesoscopic variables in given time window
  - Discover slow time evolution of effective variables by sliding time window
- Time Series Segmentation
  - Discover number/type of macroscopic phases
  - Discover lifetimes of macroscopic phases
  - Discover time scales of transitions between macroscopic phases

Modeling Nonstationary Time Series

- Assume non-stationary time series
  - $\mathbf{x} = (x_1, x_2, \ldots, x_N)$
  - M stationary segments
  - In segment m, data points drawn from (μ_m, σ_m^2) Gaussian distribution
- Recursive segmentation
  - One time series → two segments
  - Each segment → two subsegments
  - Iterate + optimize
  - Terminate

Jensen-Shannon Divergence

- Single-segment likelihood for $\mathbf{x} = (x_1, x_2, \ldots, x_N)$

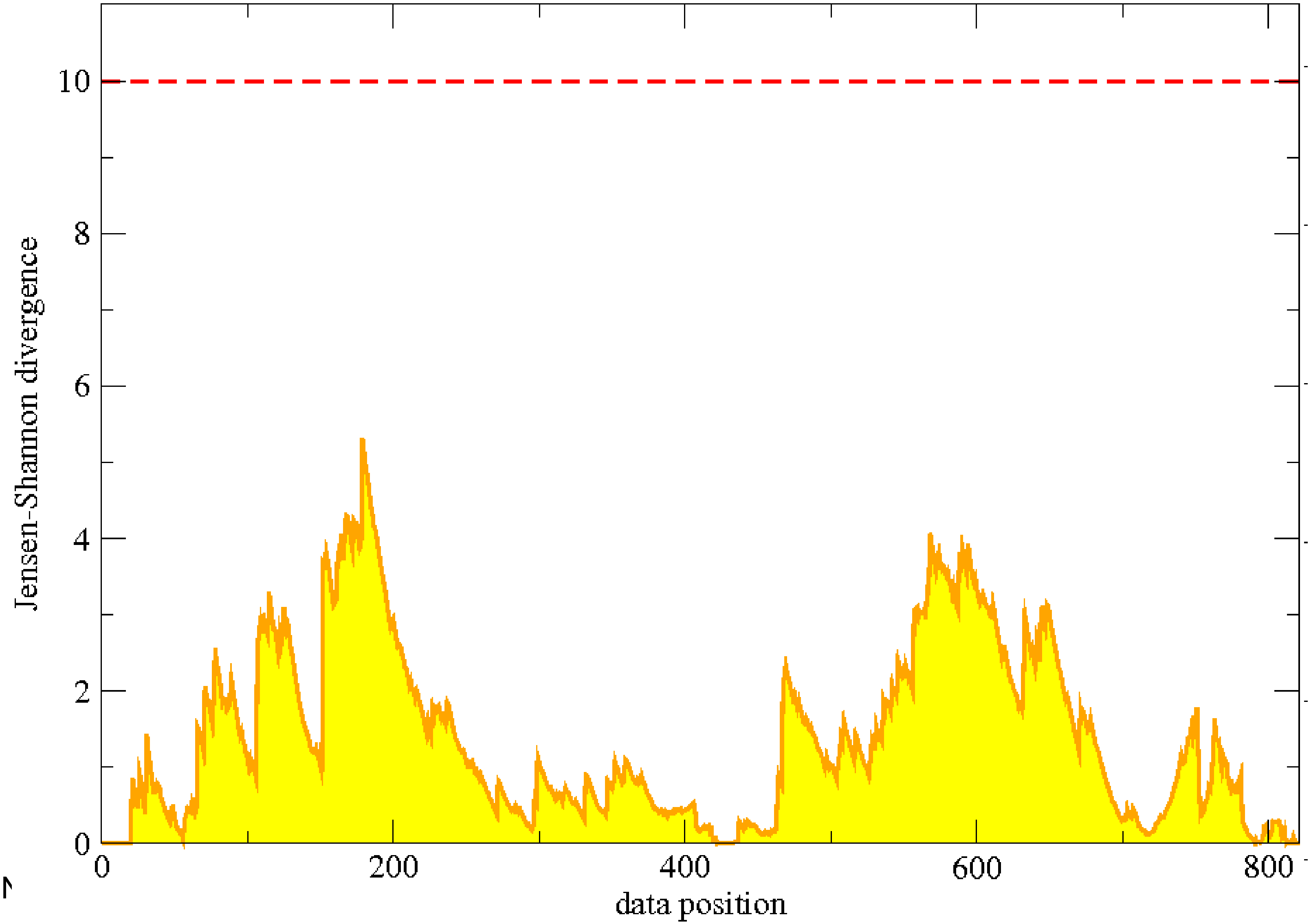
$$L_1 = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x_i - \mu)^2}{2\sigma^2}\right]$$

- Two-segment likelihood for $\mathbf{x} = (x_1, \ldots, x_t, x_{t+1}, \ldots, x_N)$

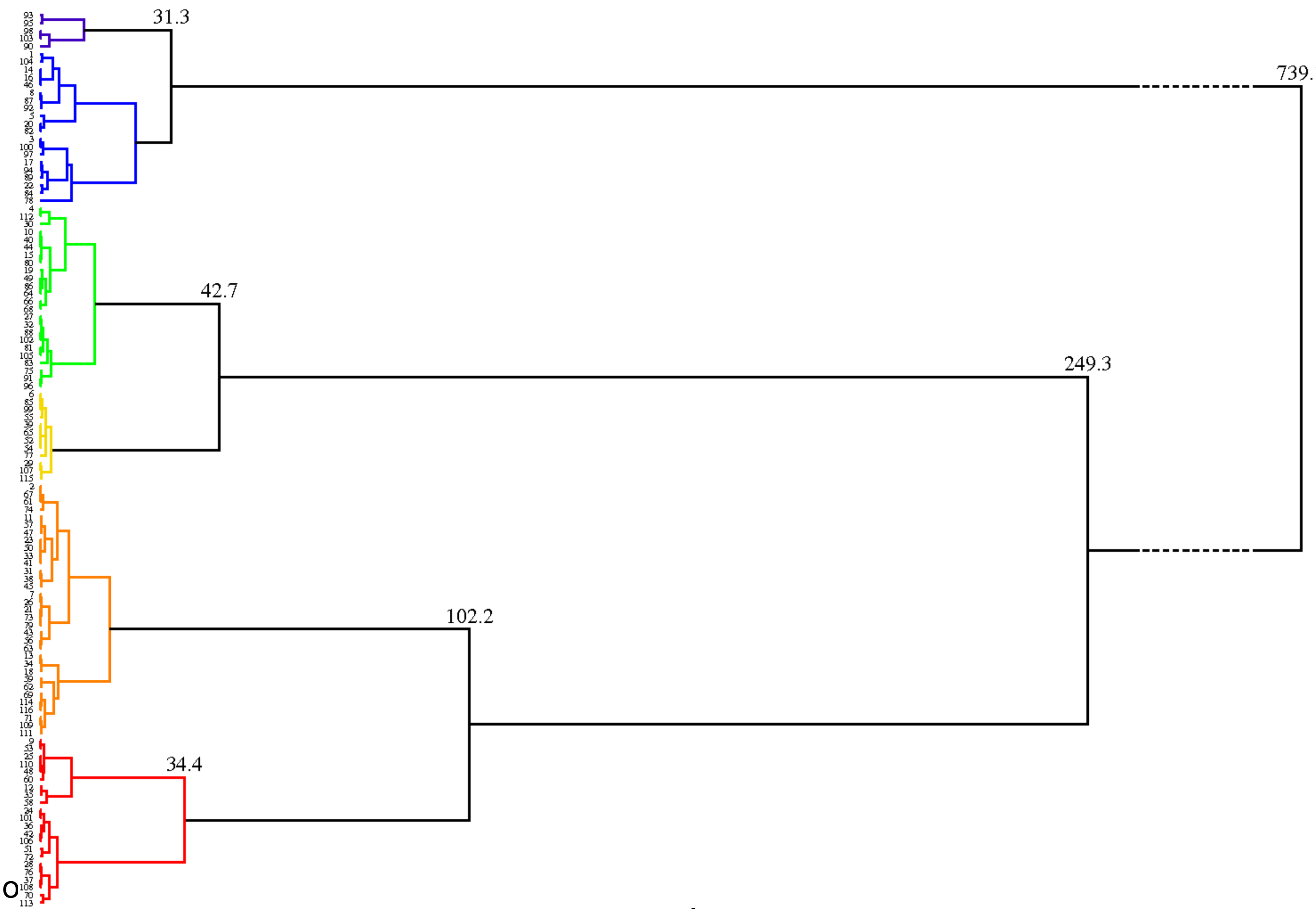
$$L_2(t) = \prod_{i=1}^{t} \frac{1}{\sqrt{2\pi\sigma_L^2}} \exp\left[-\frac{(x_i - \mu_L)^2}{2\sigma_L^2}\right] \prod_{i=t+1}^{N} \frac{1}{\sqrt{2\pi\sigma_R^2}} \exp\left[-\frac{(x_i - \mu_R)^2}{2\sigma_R^2}\right]$$

- ML estimates $\hat{\mu}, \hat{\mu}_L, \hat{\mu}_R, \hat{\sigma}^2, \hat{\sigma}_L^2, \hat{\sigma}_R^2$
- Jensen-Shannon divergence $\Delta(t) = \ln \dfrac{L_2(t)}{L_1} \geq 0$

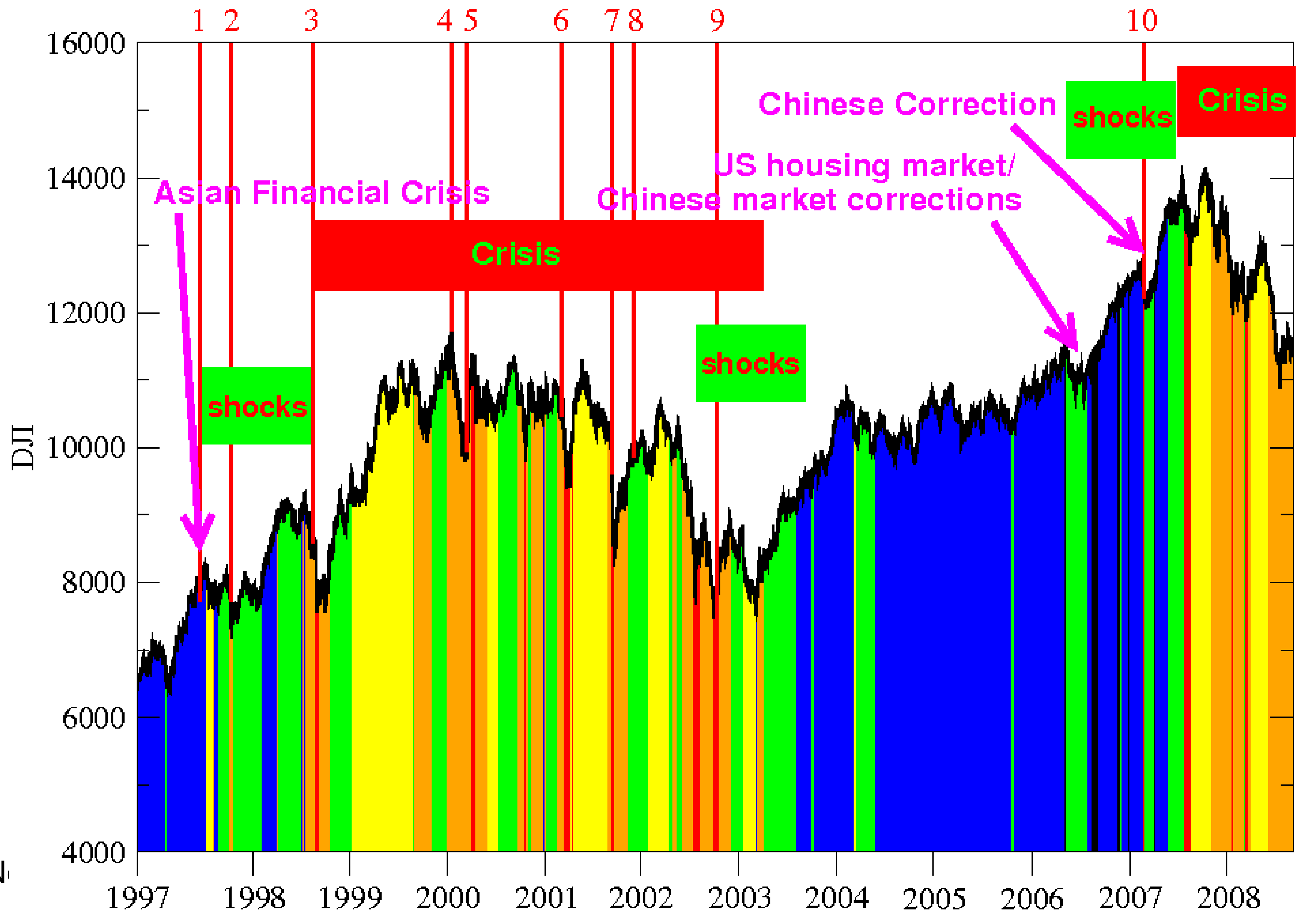
Recursive Segmentation



Segment Clustering



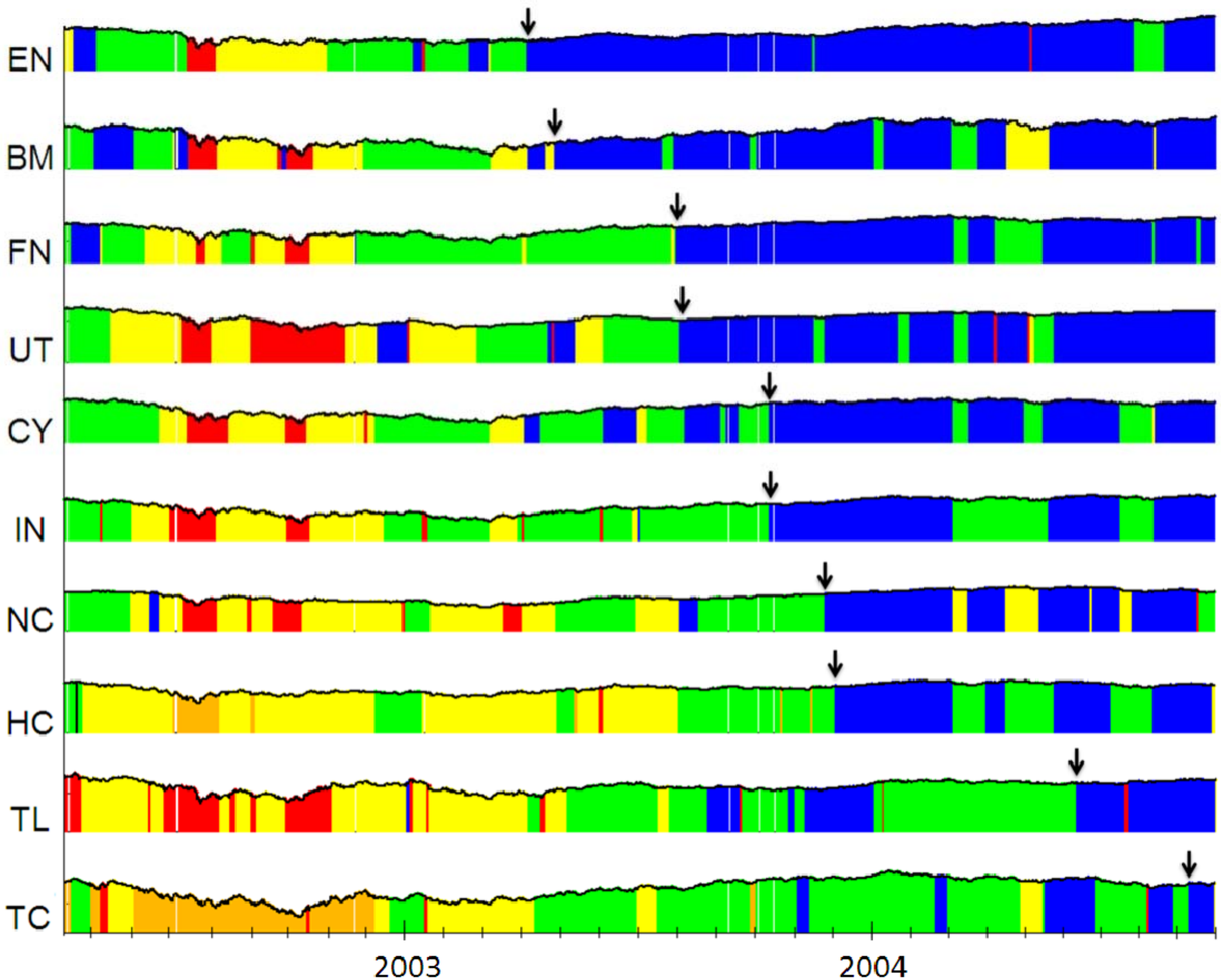
Temporal Distribution of Segments



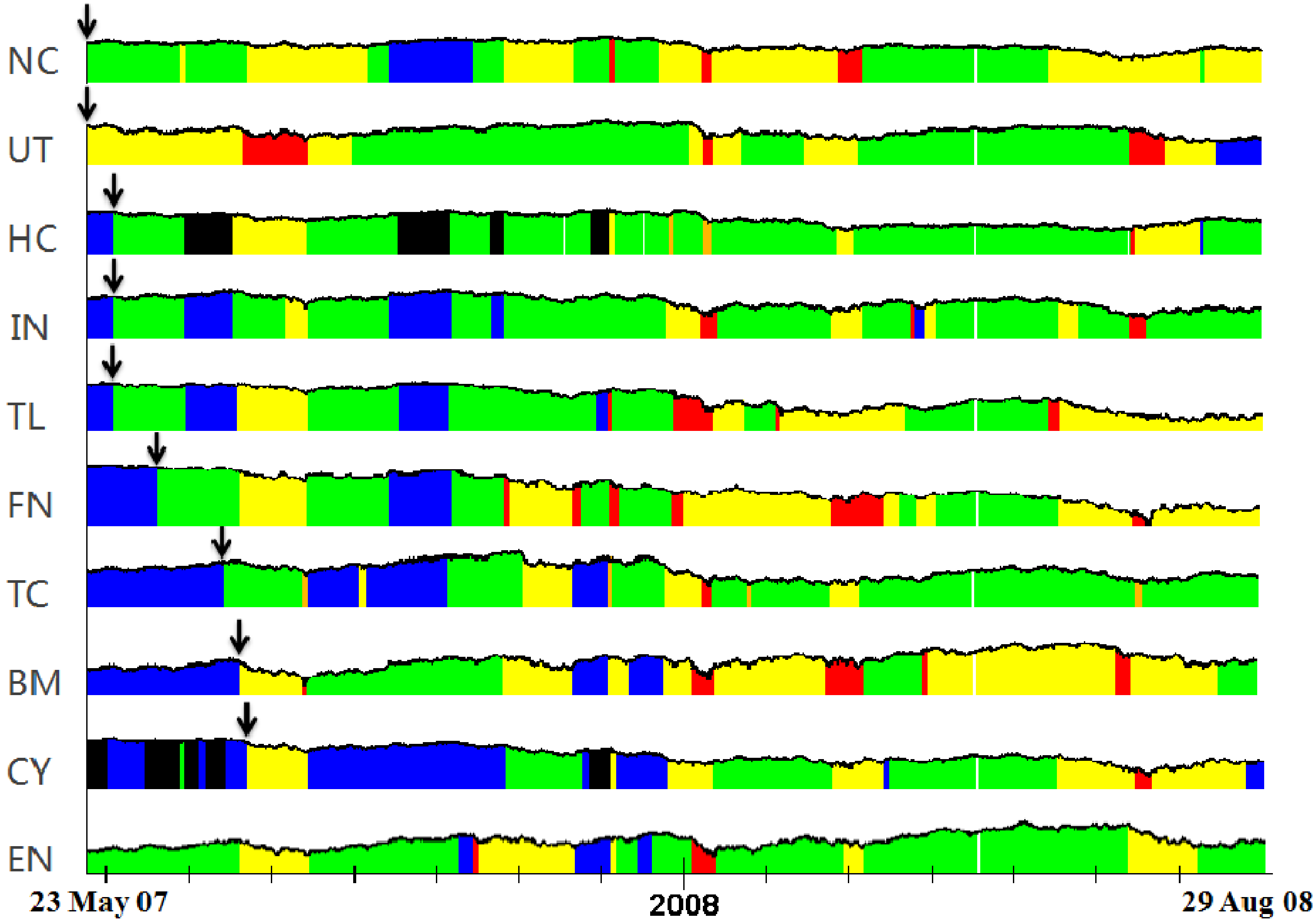
Cross Section Study

k	*Symbol*	*Economic Sector*
1	BM	Basic Materials
2	CY	Consumer Services
3	EN	Oil & Gas
4	FN	Financials
5	HC	Healthcare
6	IN	Industrials
7	NC	Consumer Goods
8	TC	Technologies
9	TL	Telecommunications
10	UT	Utilities

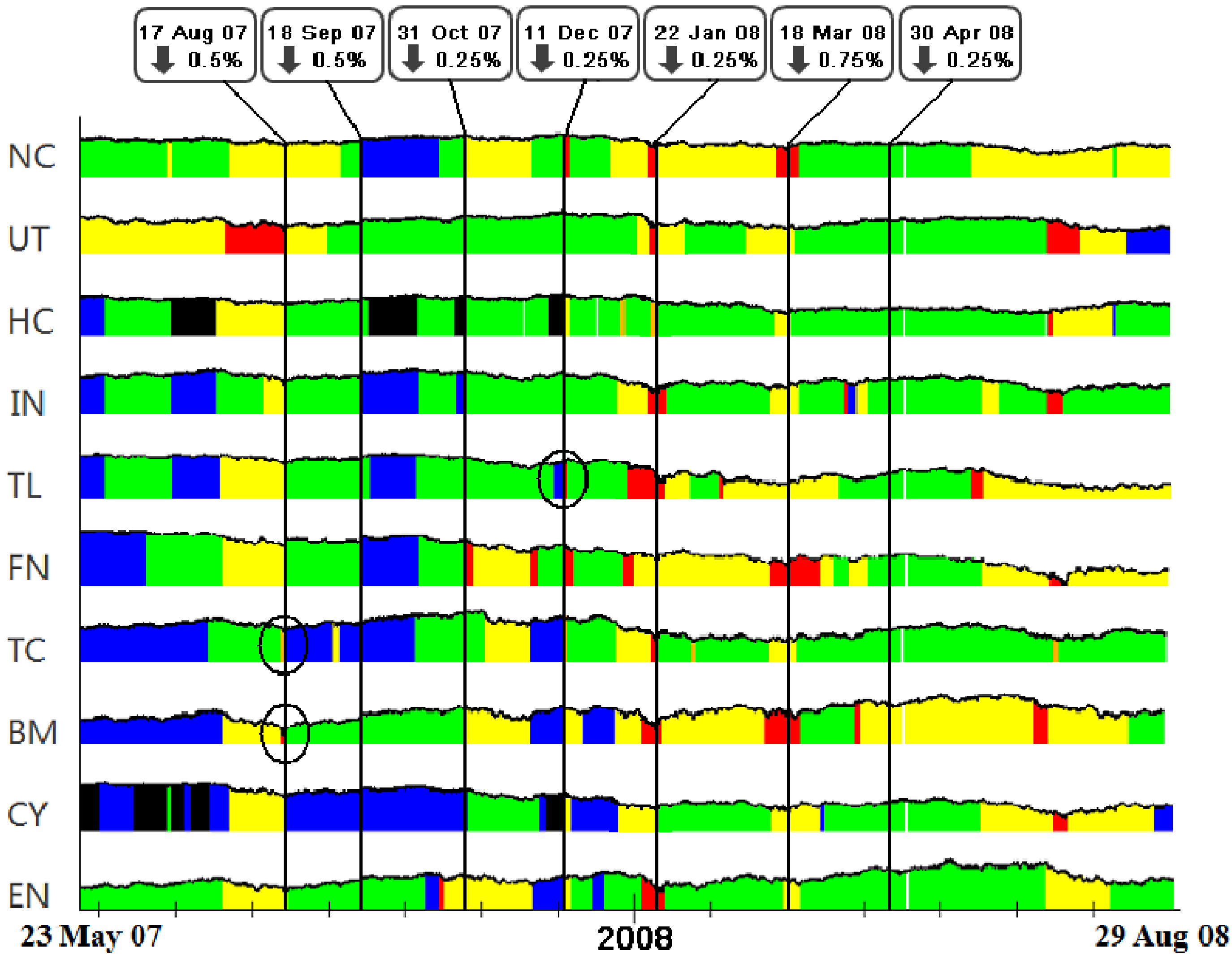
Economic Recovery



Onset of Crisis



Response to Extrinsic Shocks



Conclusions

- Potential for understanding macroscopic earthquake dynamics from microscopic time series
- Time series clustering
  - Complete separation of scales
    - Metallic nanoclusters
  - Partial separation of scales
    - New Zealand 2007 earthquake precursor dynamics
- Time series segmentation
  - US economy
    - Crisis and growth
    - Chemical picture of slow dynamics using MSTs

Acknowledgments

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  - Prof Lai S K
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Thank You!