Understanding Multi-Key Security Degradation

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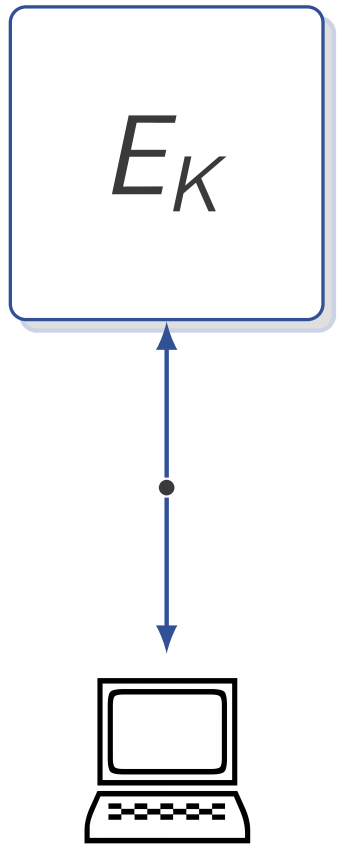
September 30, 2016

Single-Key vs. Multi-Key

1. Single-key setting usually analyzed



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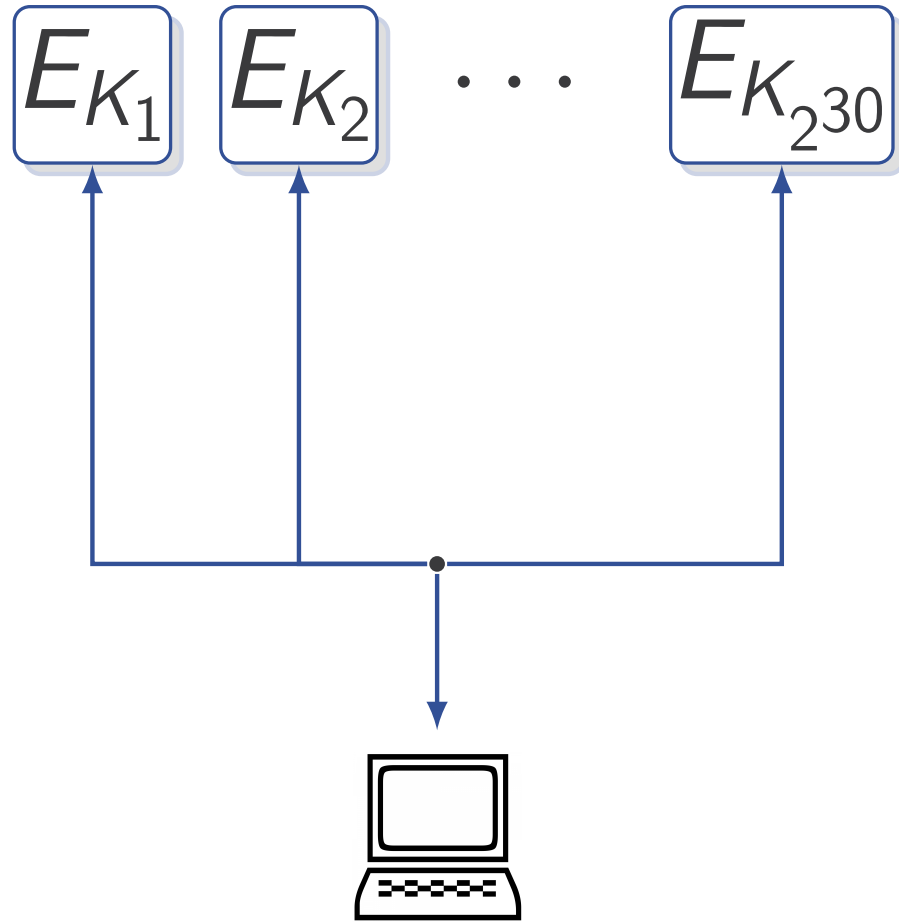
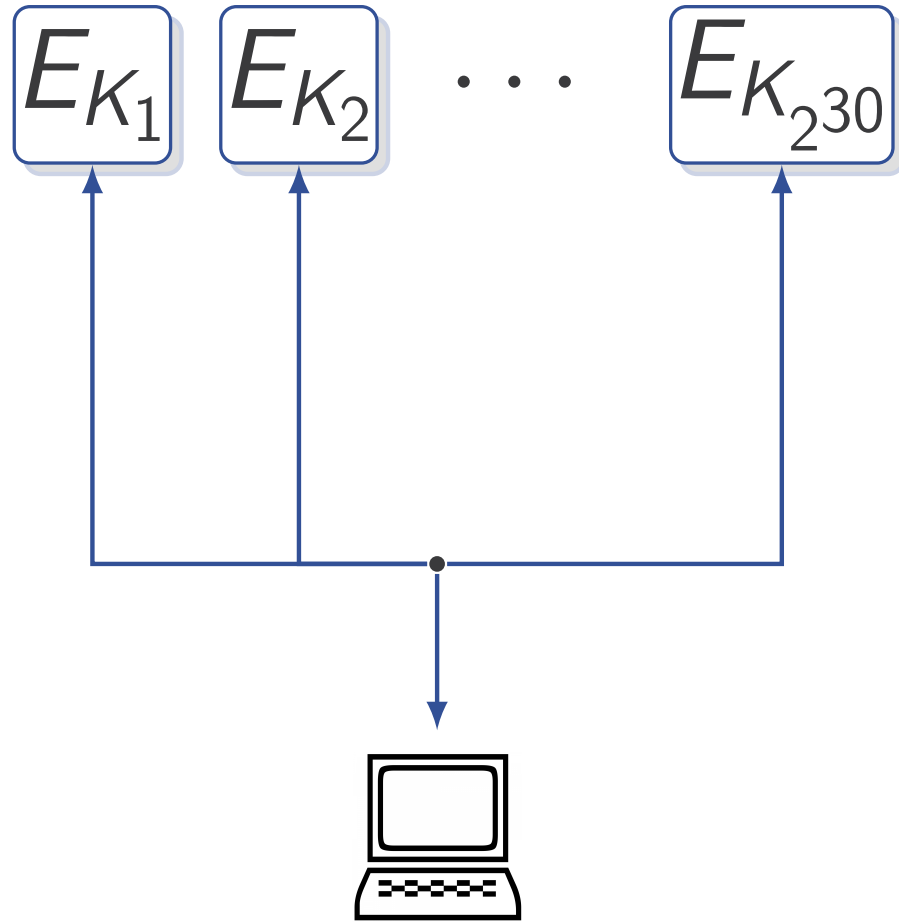
2. Multi-key setting is practically important



Single-Key vs. Multi-Key

1. Single-key setting usually analyzed
2. Multi-key setting is practically important
3. Example: AES-GCM used in TLS, hundreds of millions of keys used



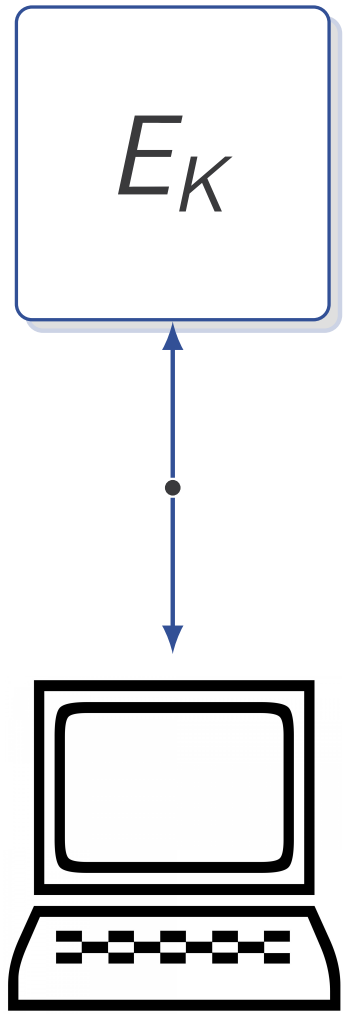
Folklore Result and Multi-Key Degradation

Multi-Key Success Probability

$\leq$

$u\times$ Single-Key Success Probability

Example : Block Ciphers



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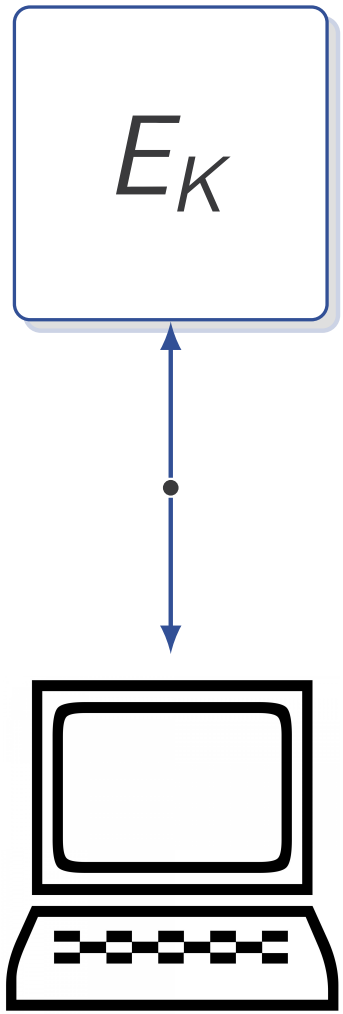

Computational complexity

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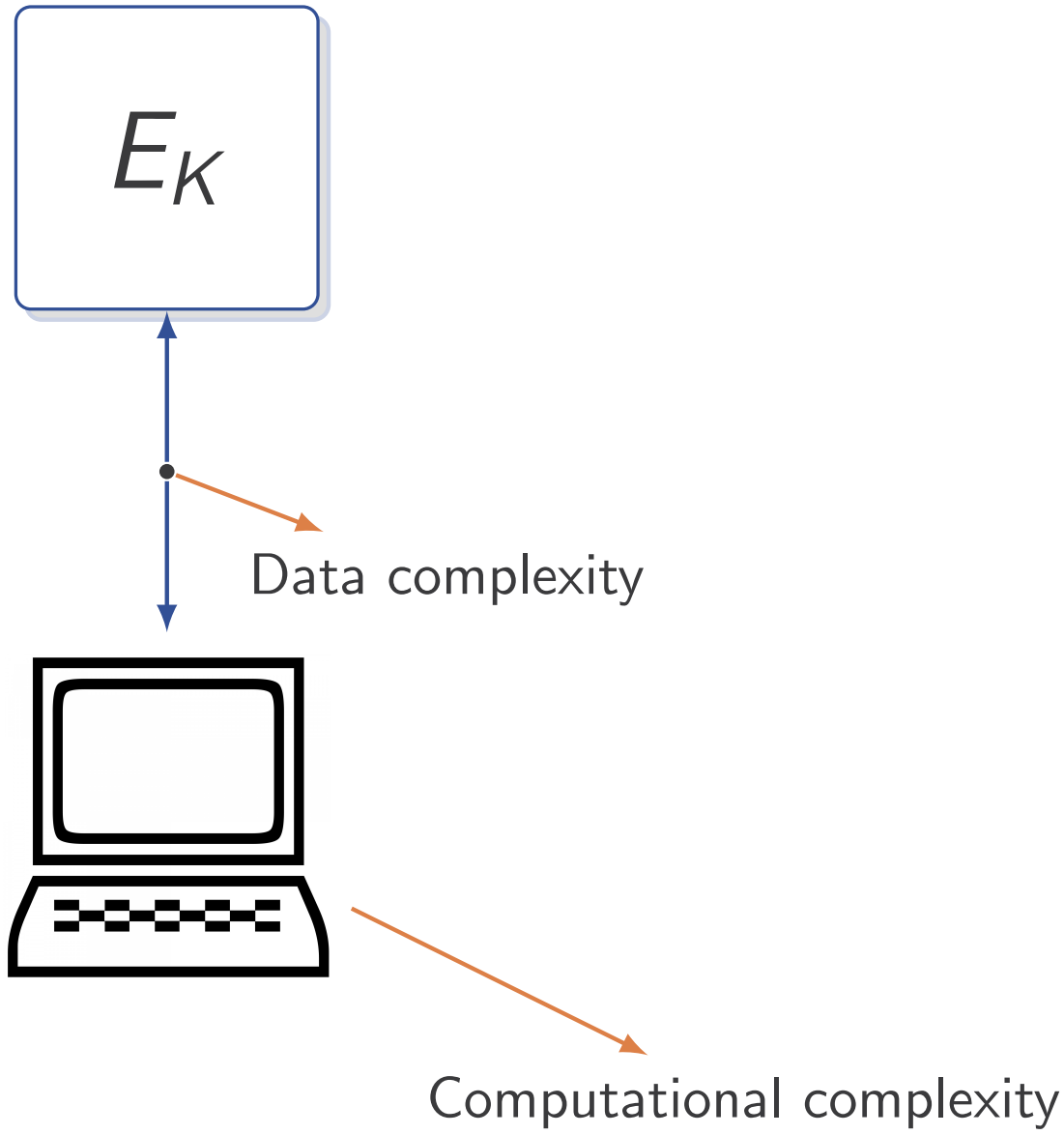

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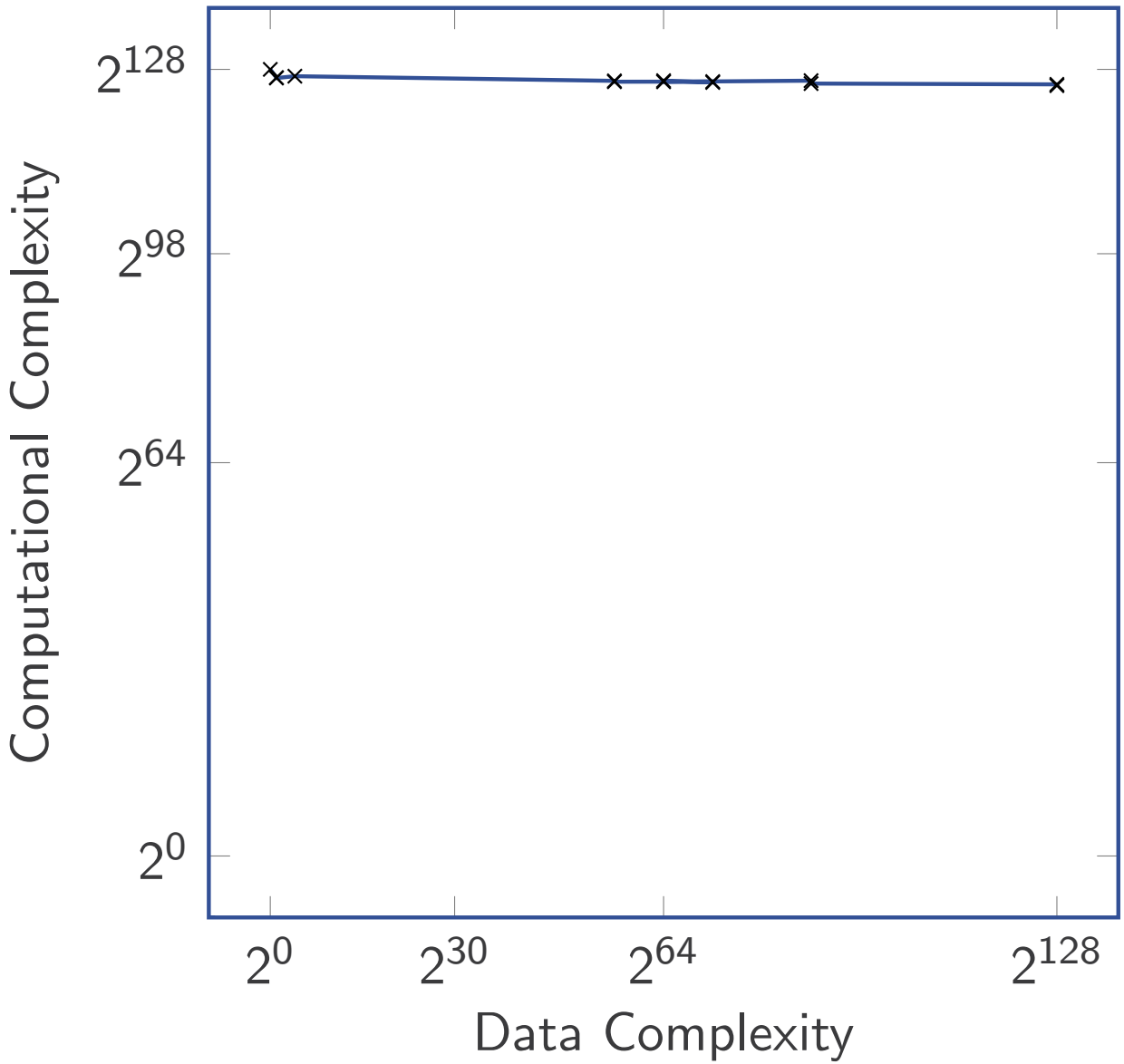

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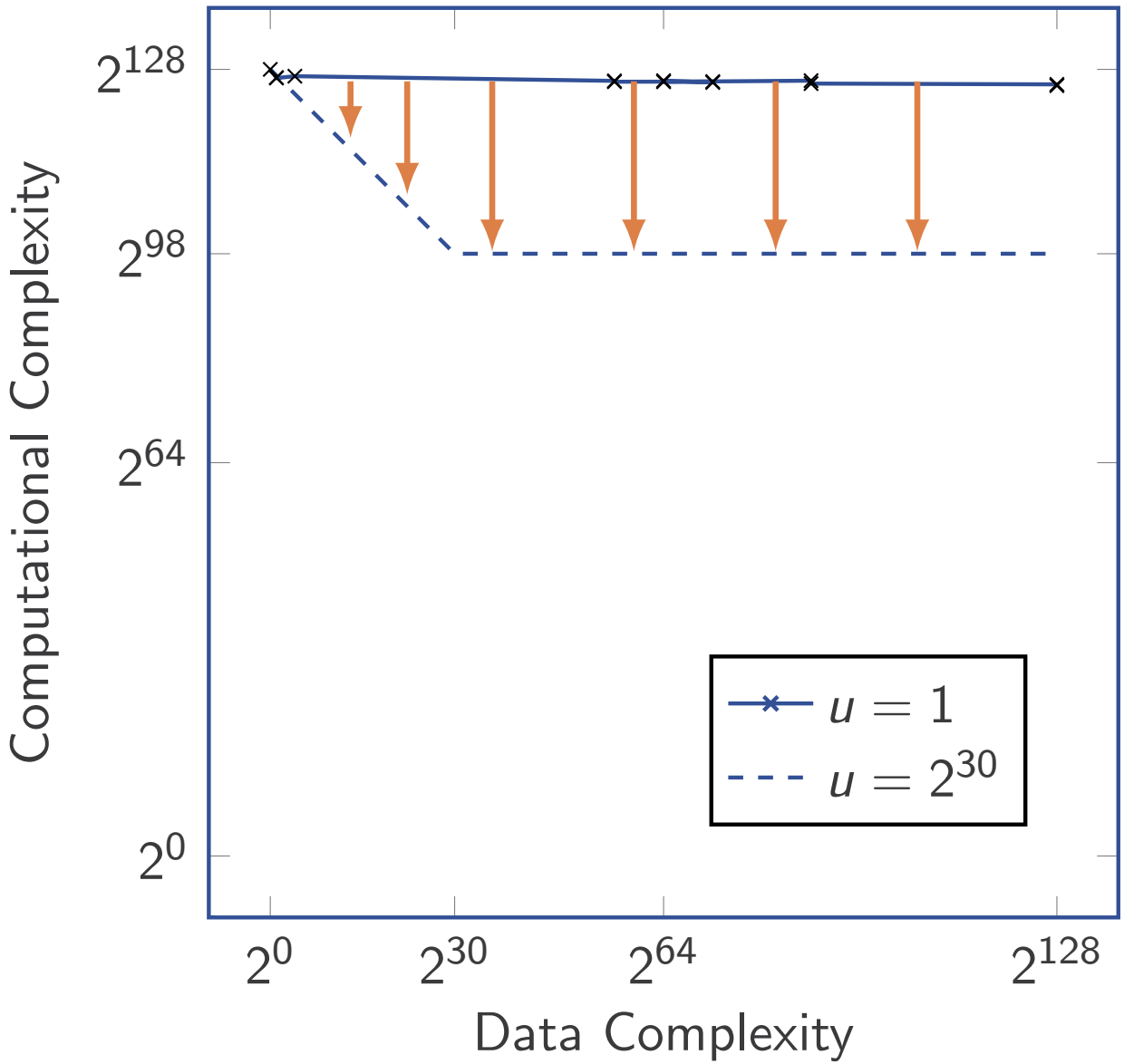

Block Cipher Multi-key Attack

Precomputation:

$(L, E_L(P))$

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Queries:

$$E_{K_1}(P), E_{K_2}(P), \ldots, E_{K_u}(P)\,.$$

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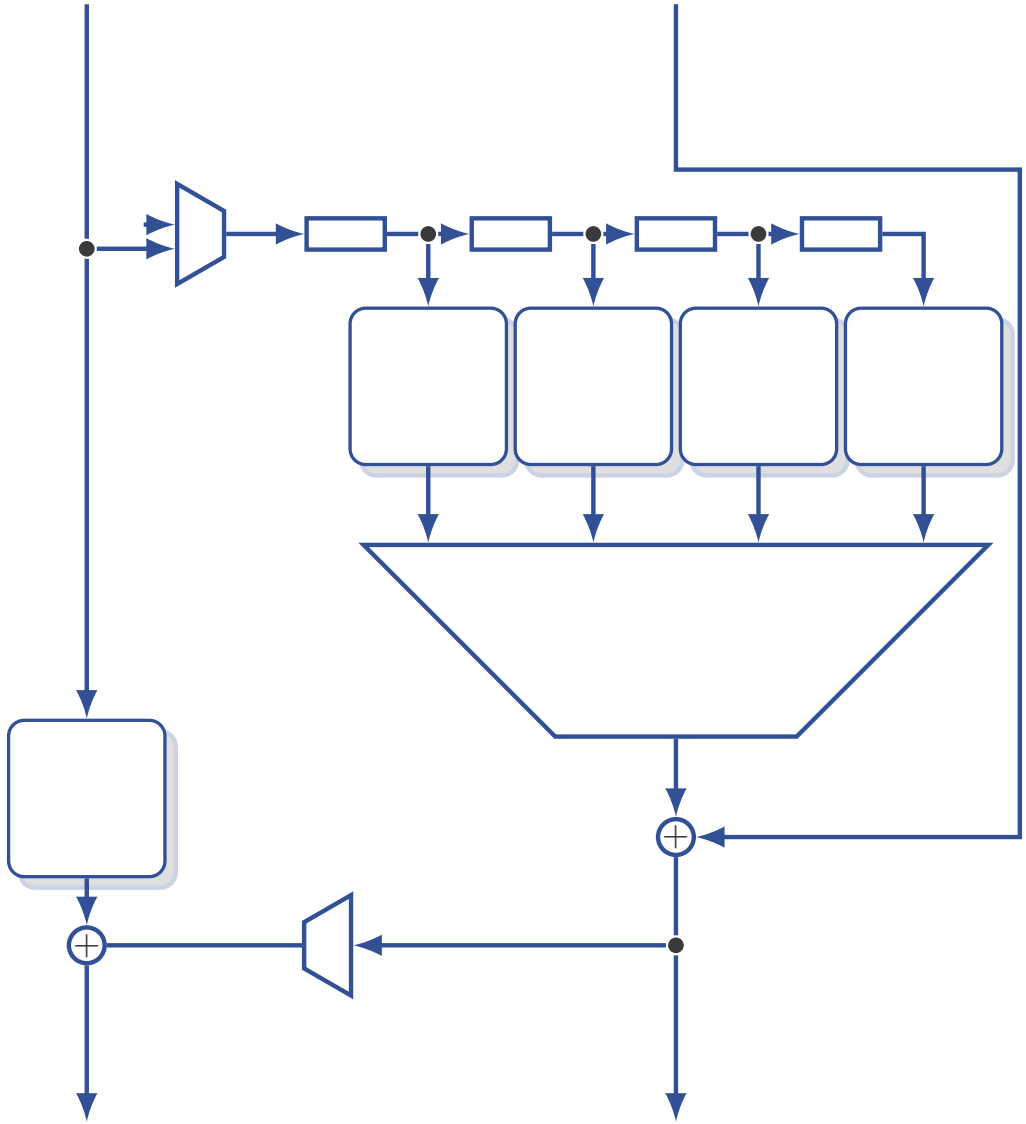
Biryukov, Mukhopadhyay, Sarkar, SAC 2005: time-memory-data trade-off

Example: GCM



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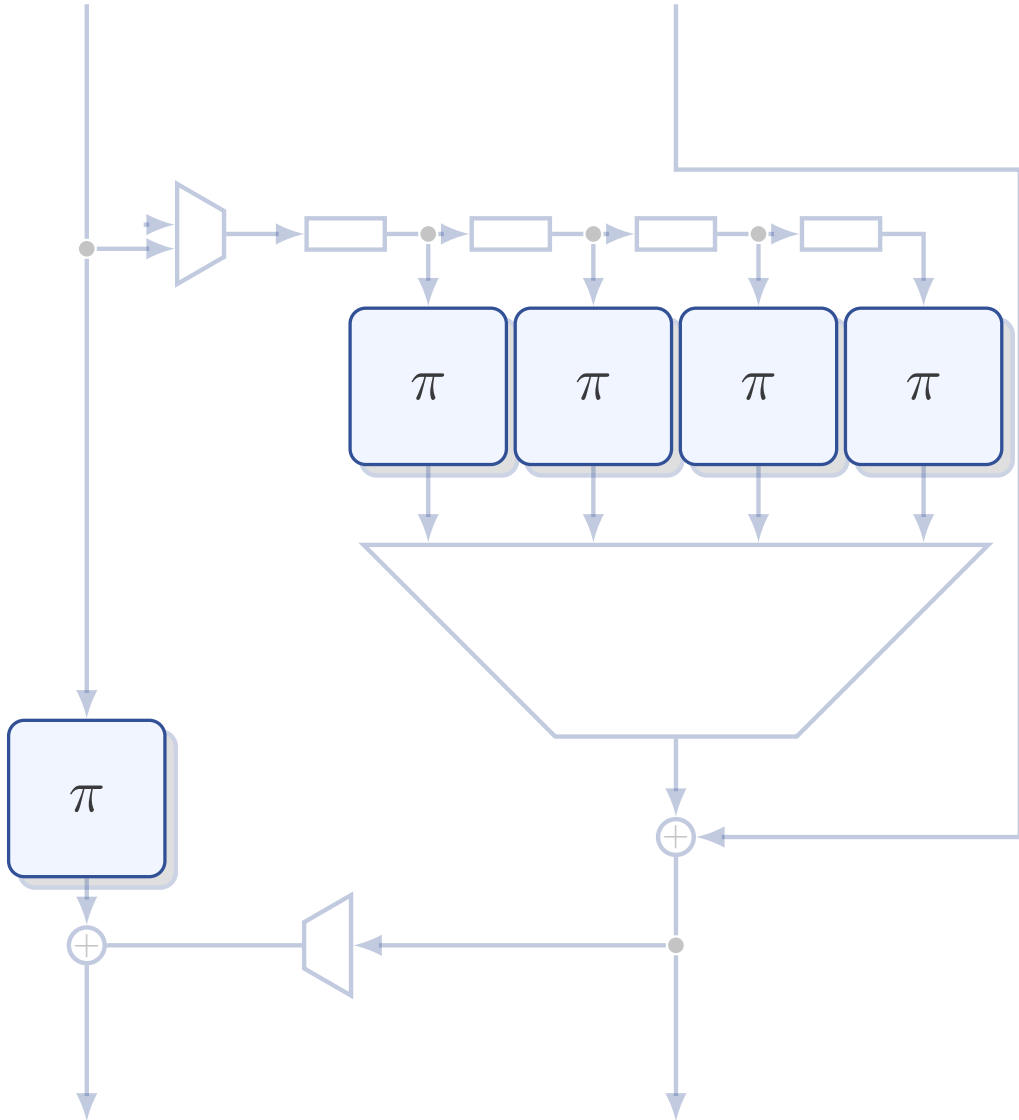
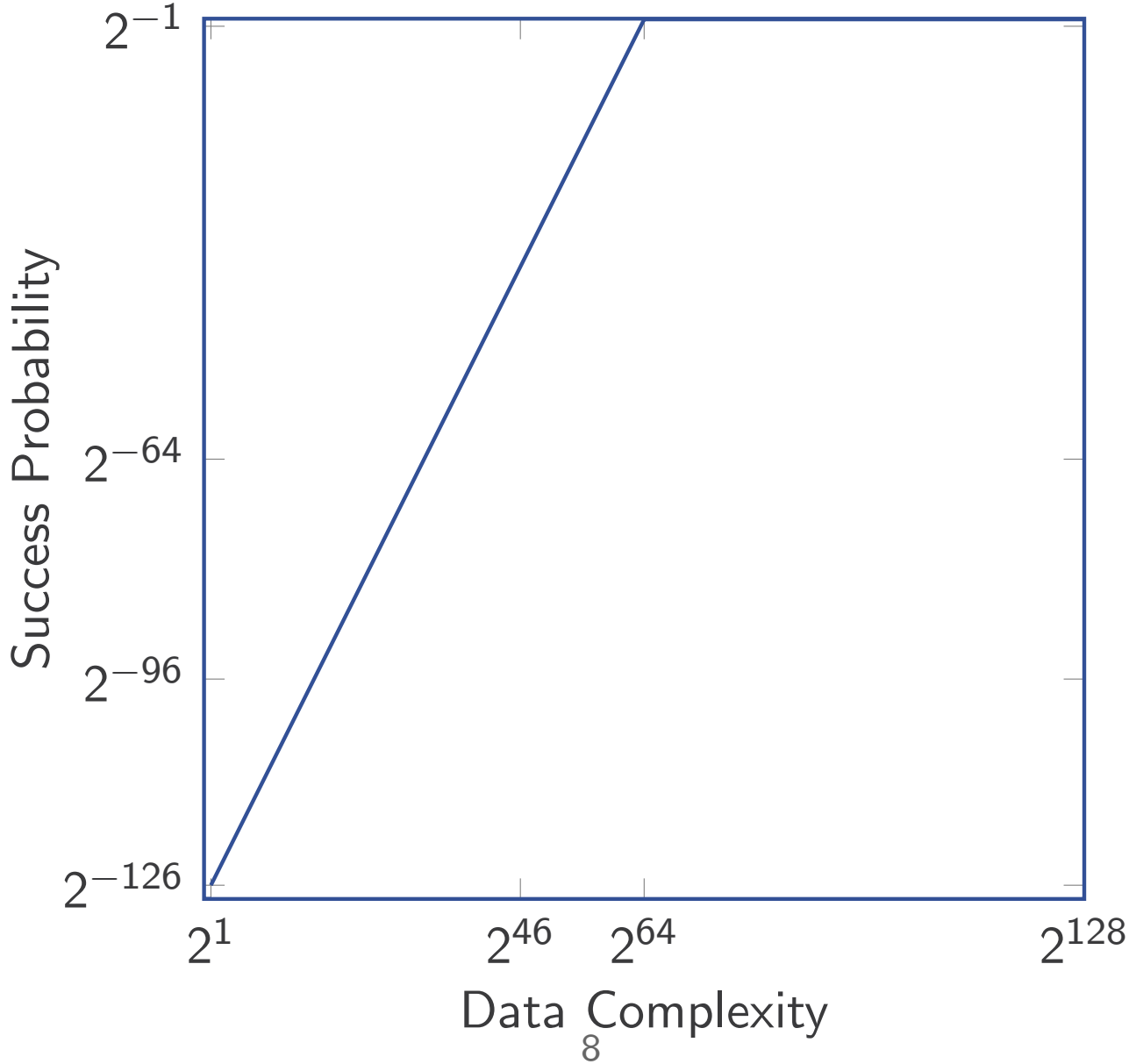


Biham attack inapplicable

GCM Bound in the Multi-Key Setting



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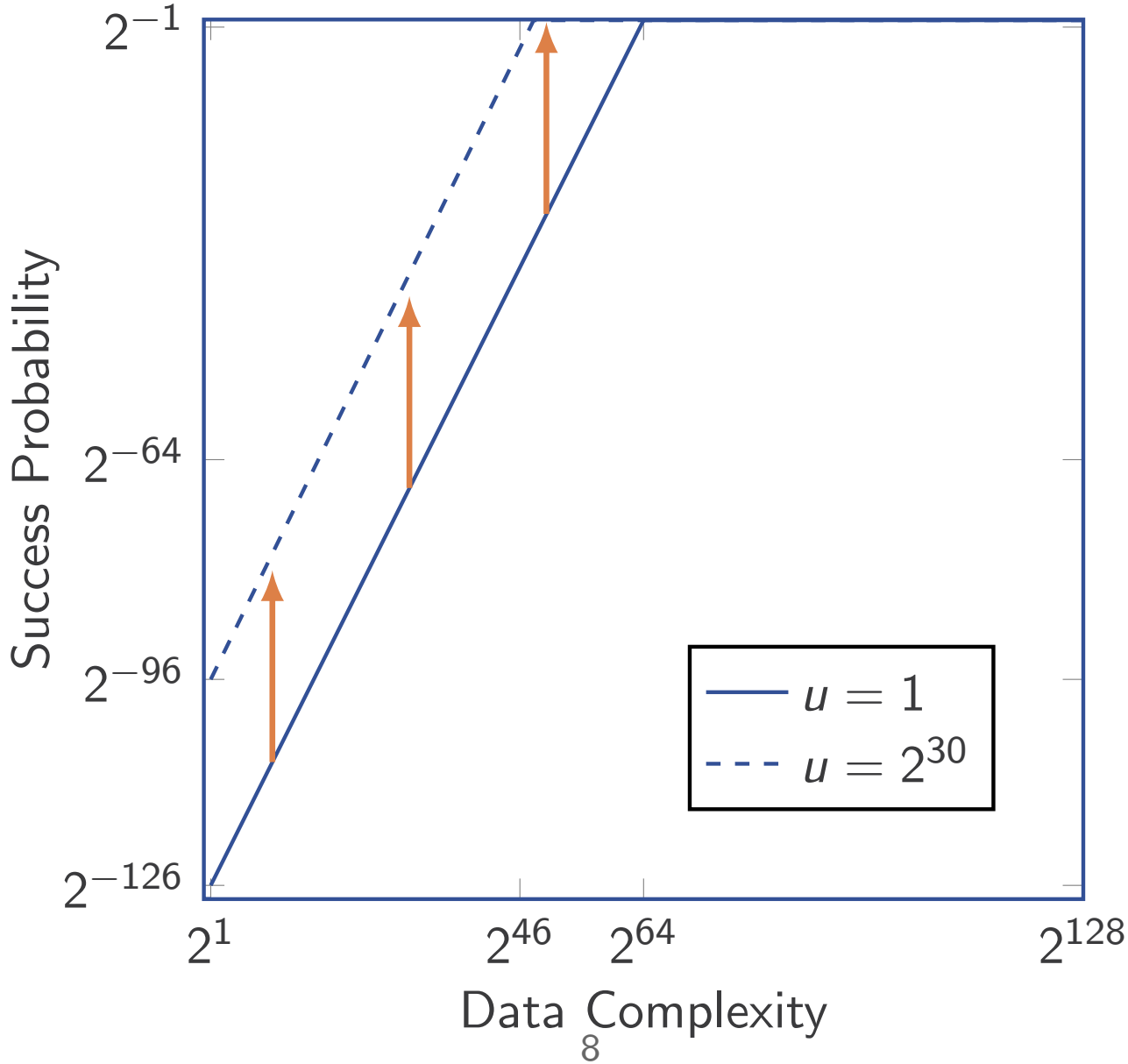
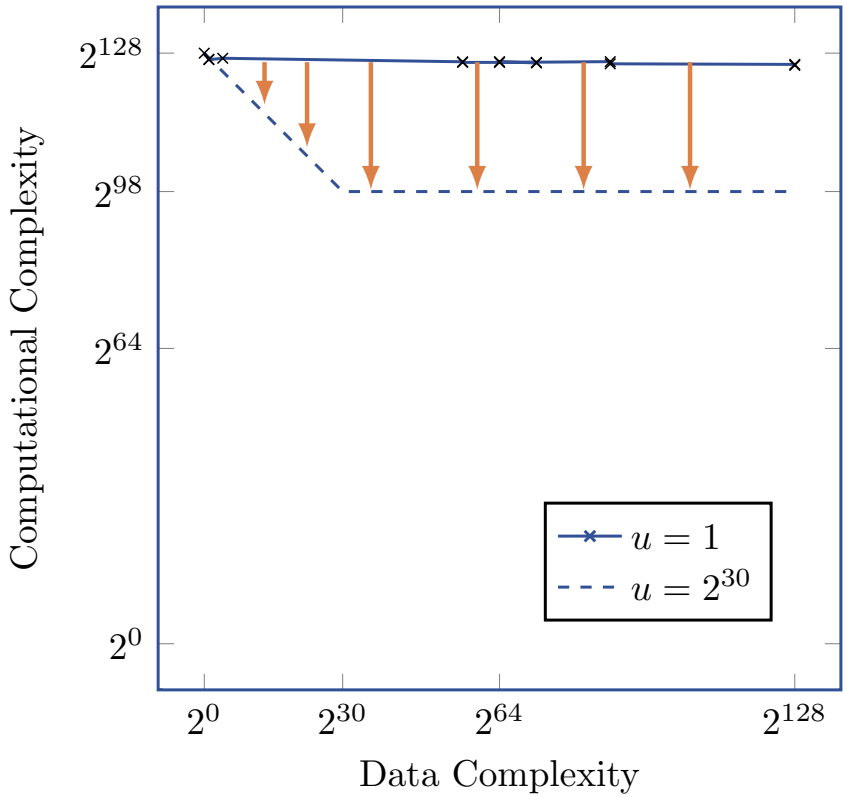
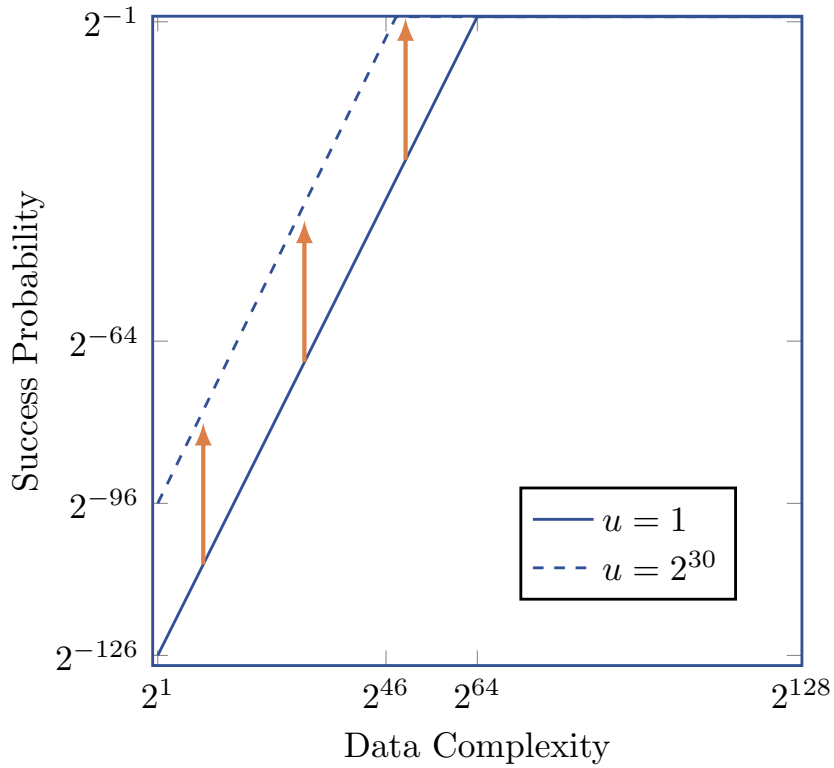


AES128 Multikey vs. GCM Multikey





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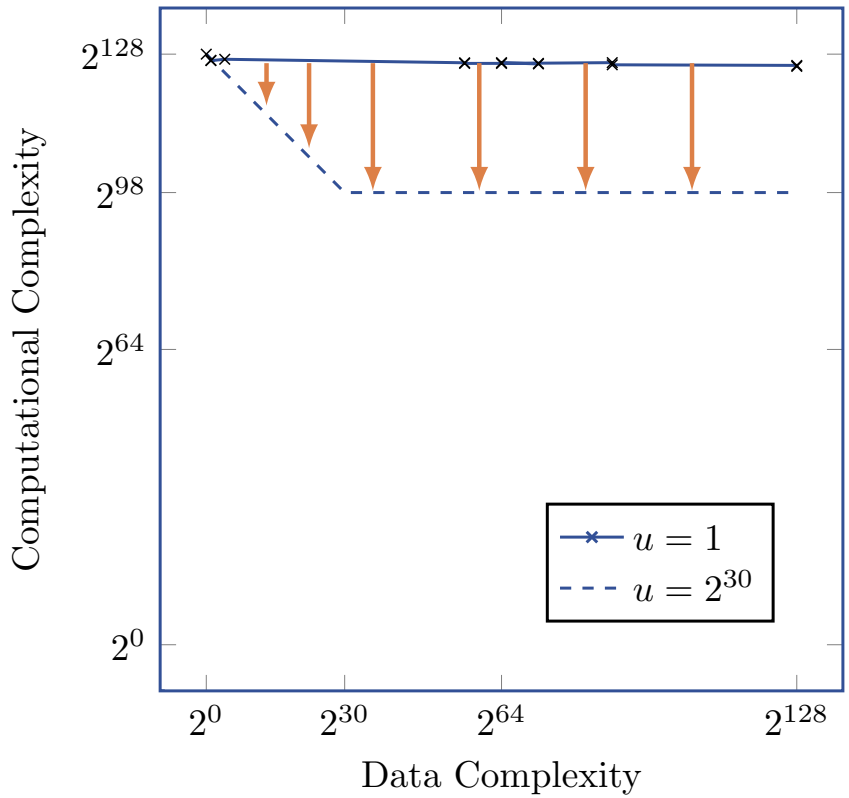
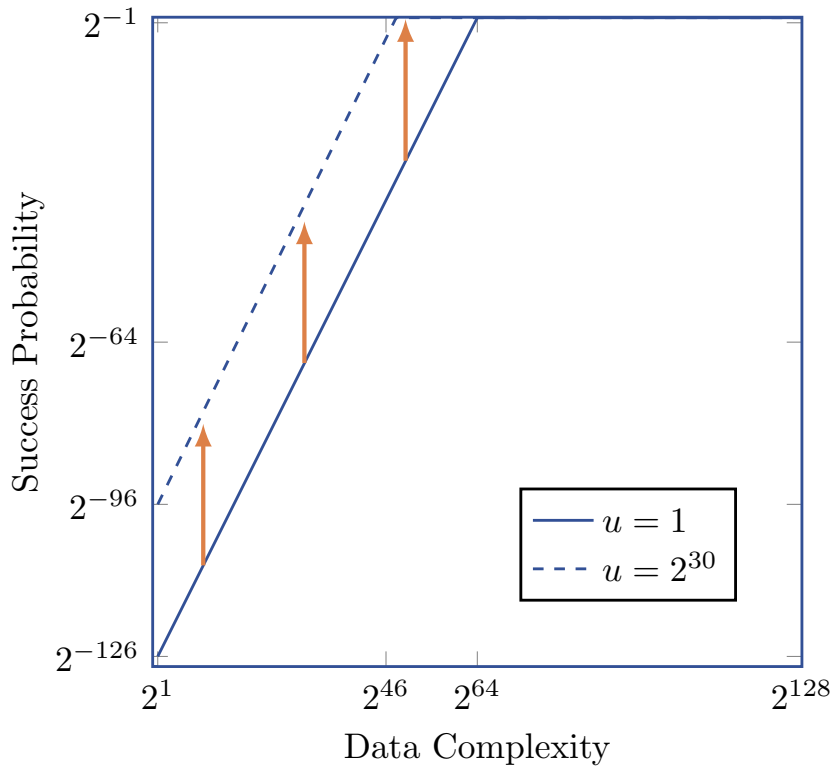
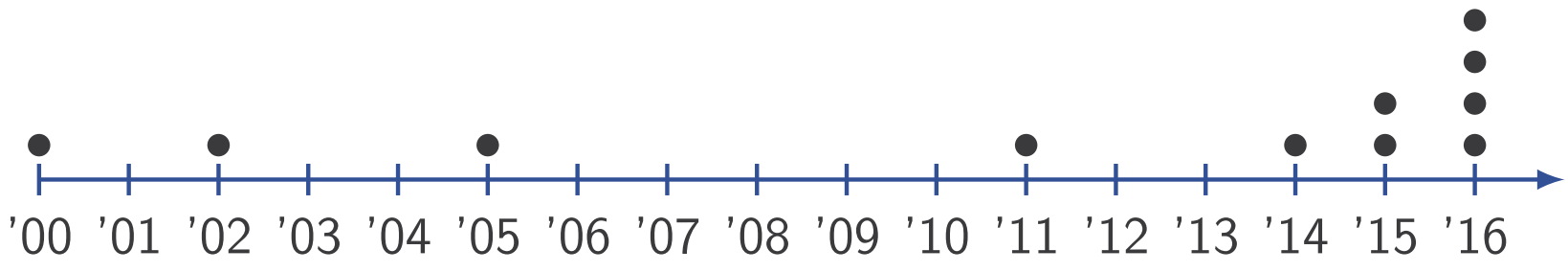




Matching attacks?

Work on Multi-Key Security in Symmetric Setting



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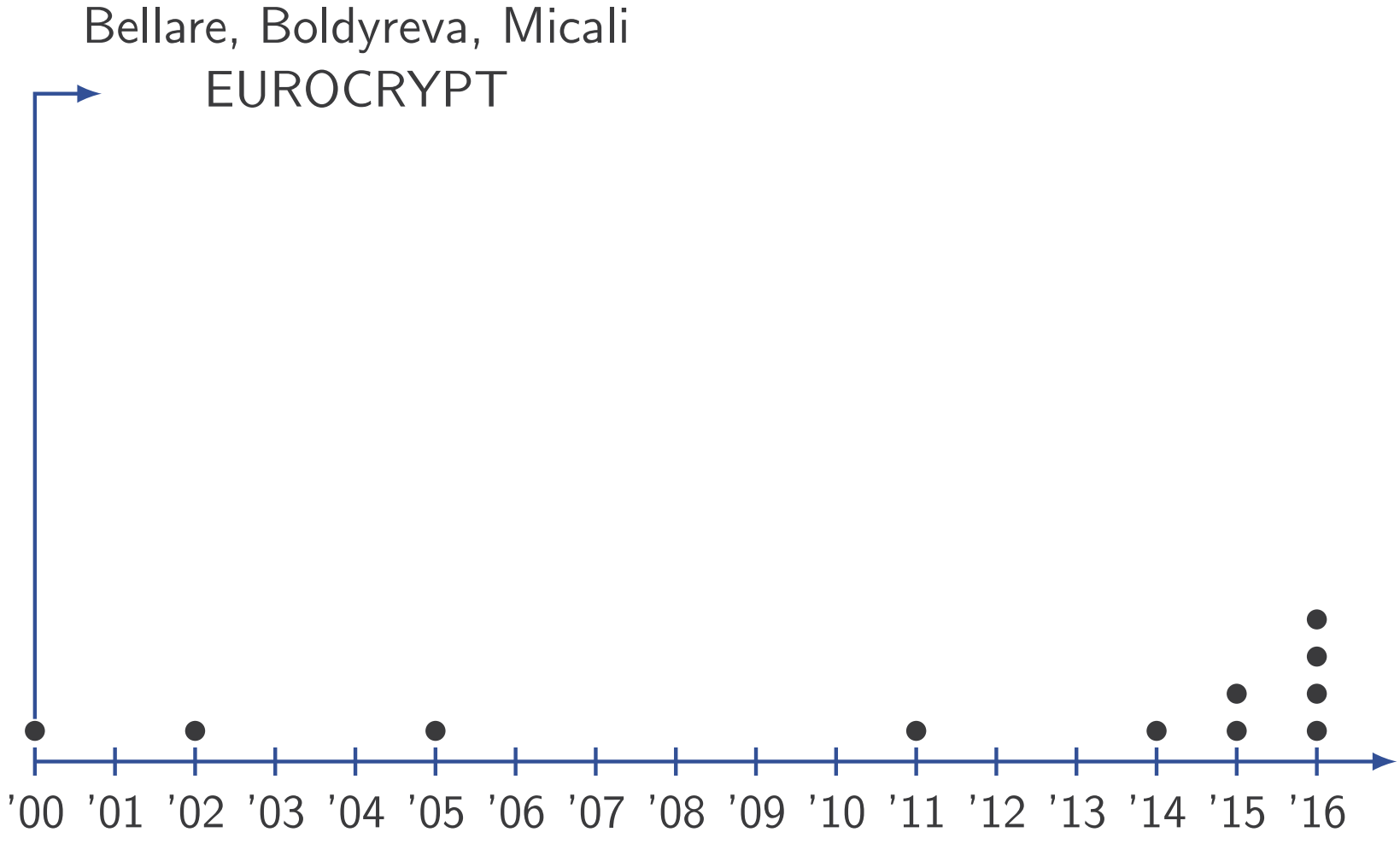

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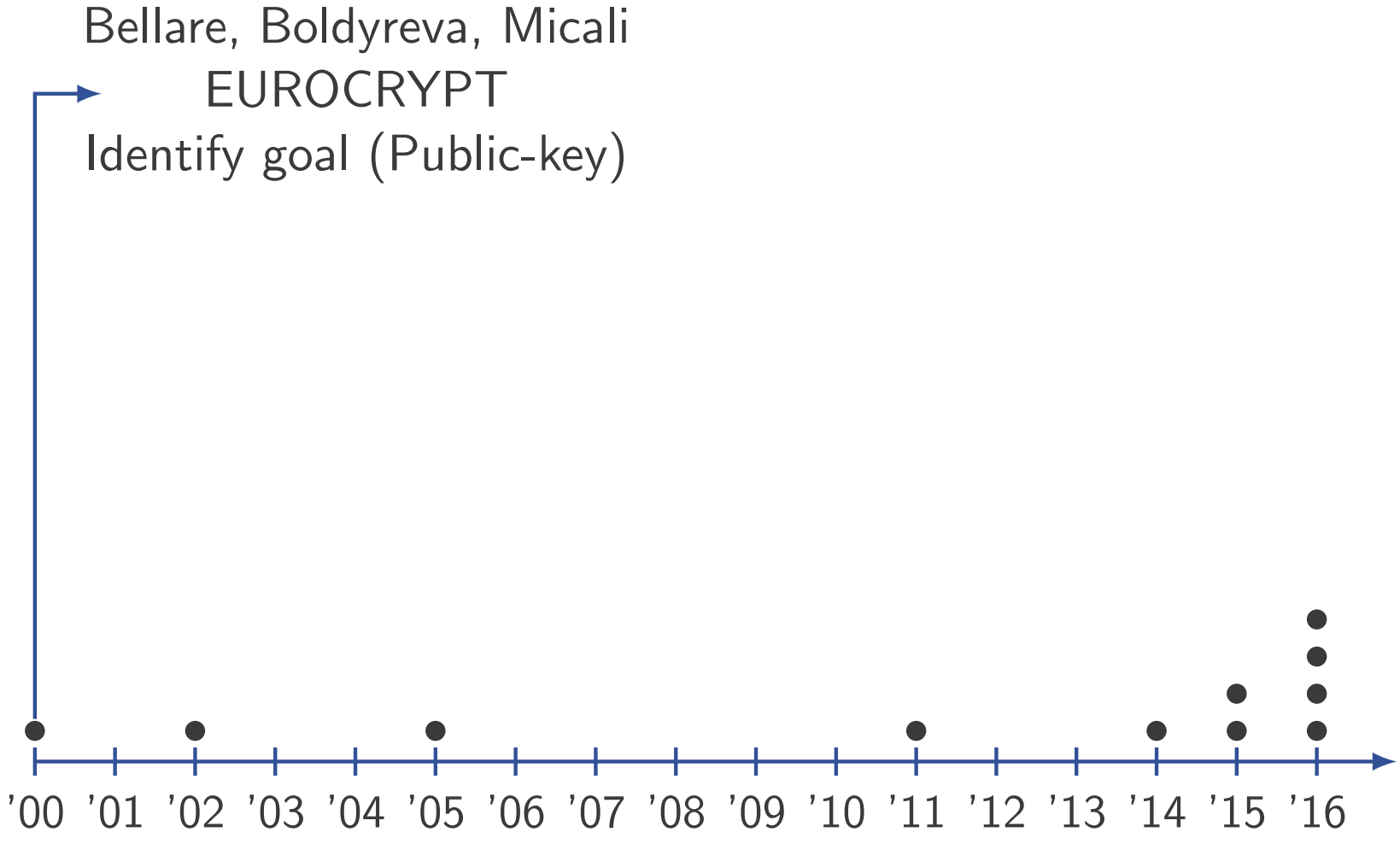

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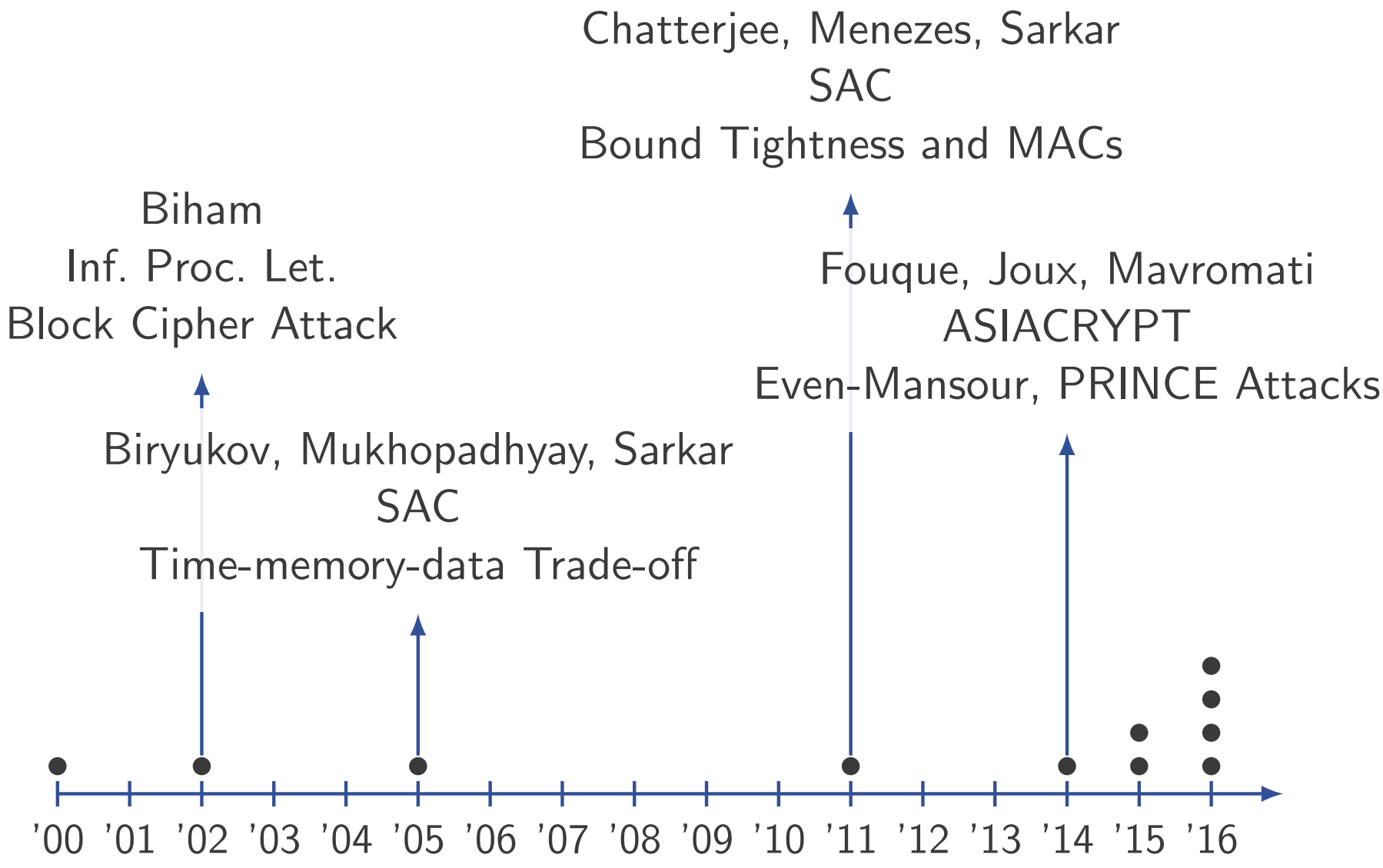

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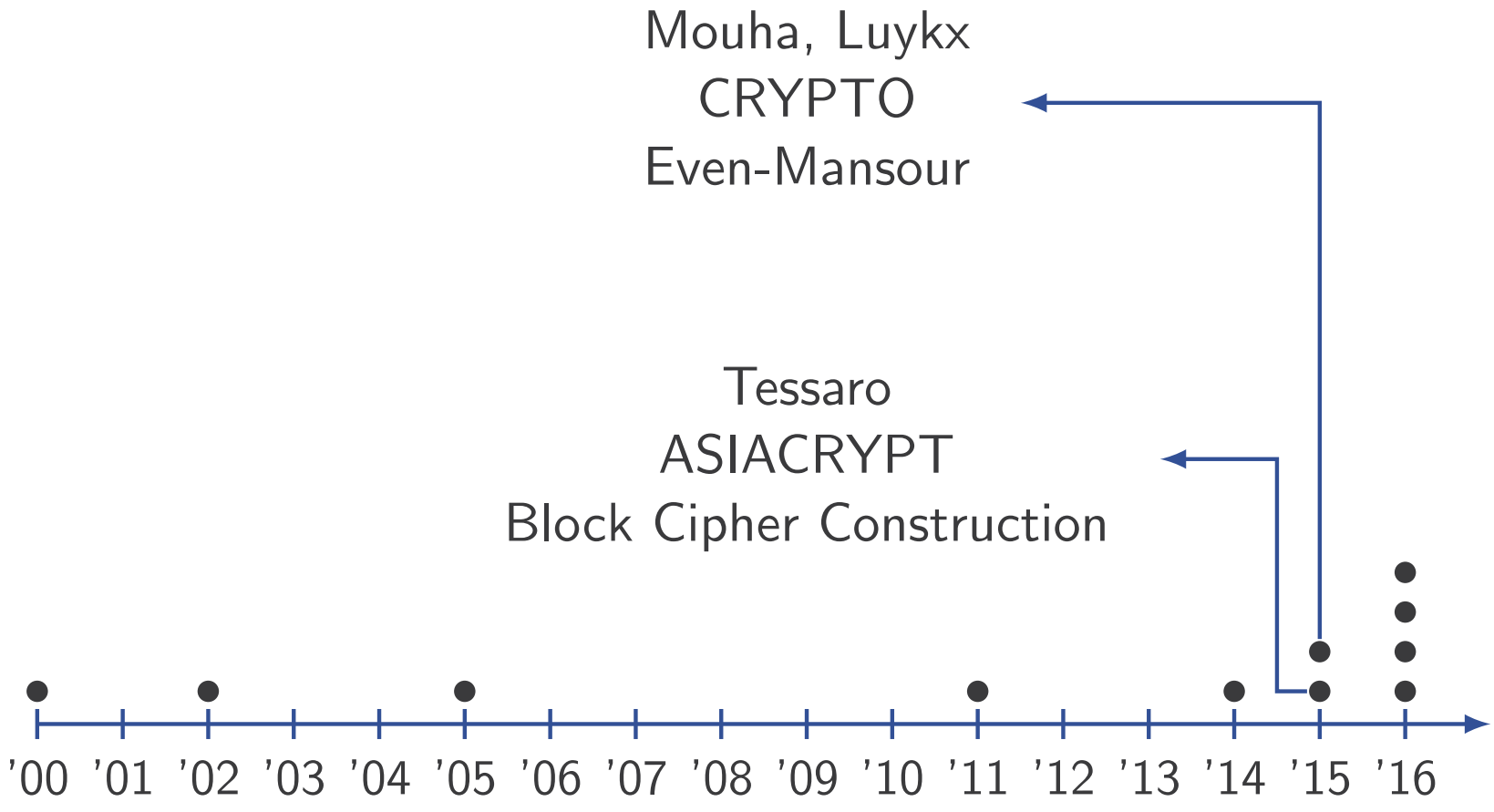

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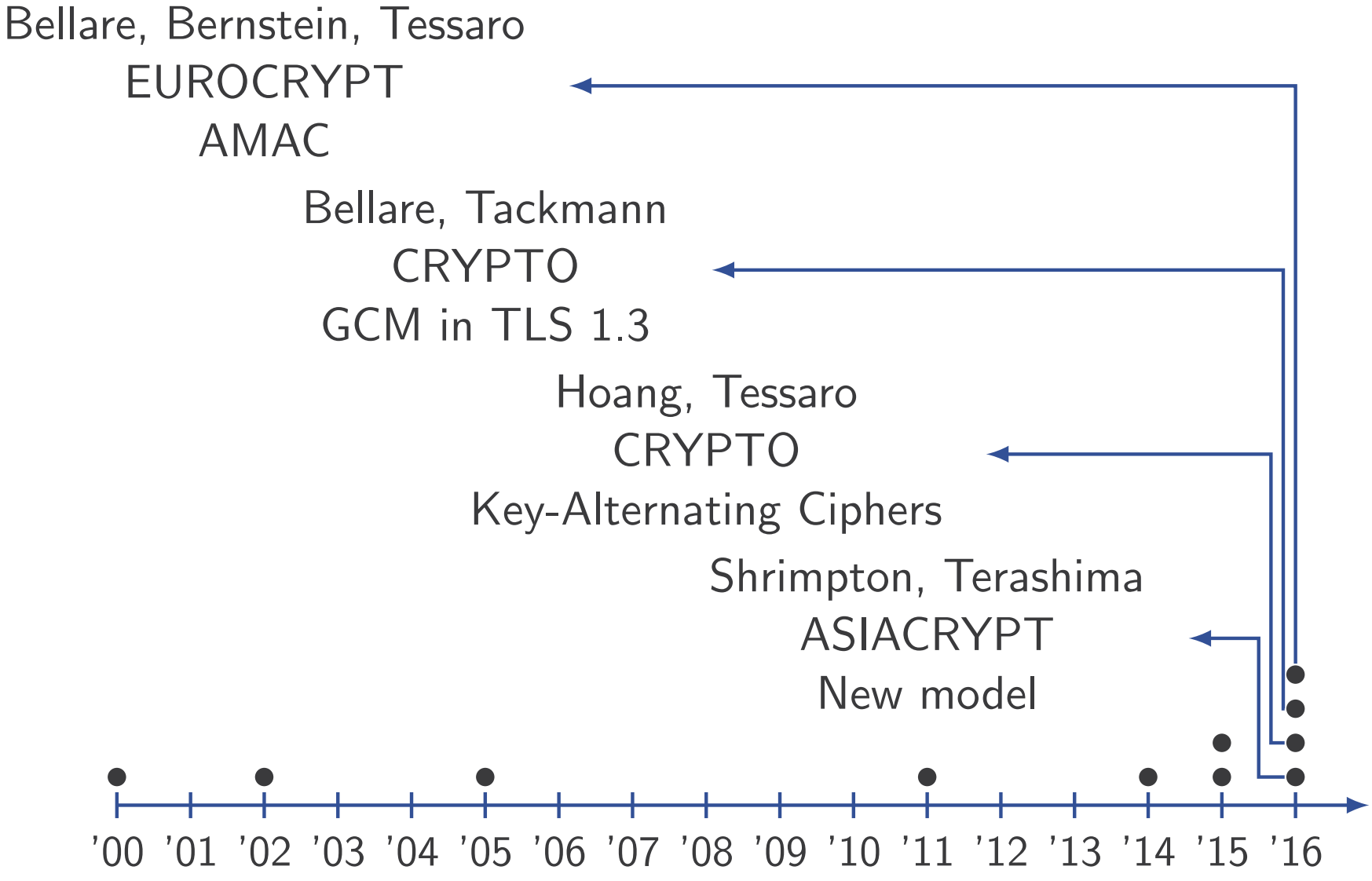
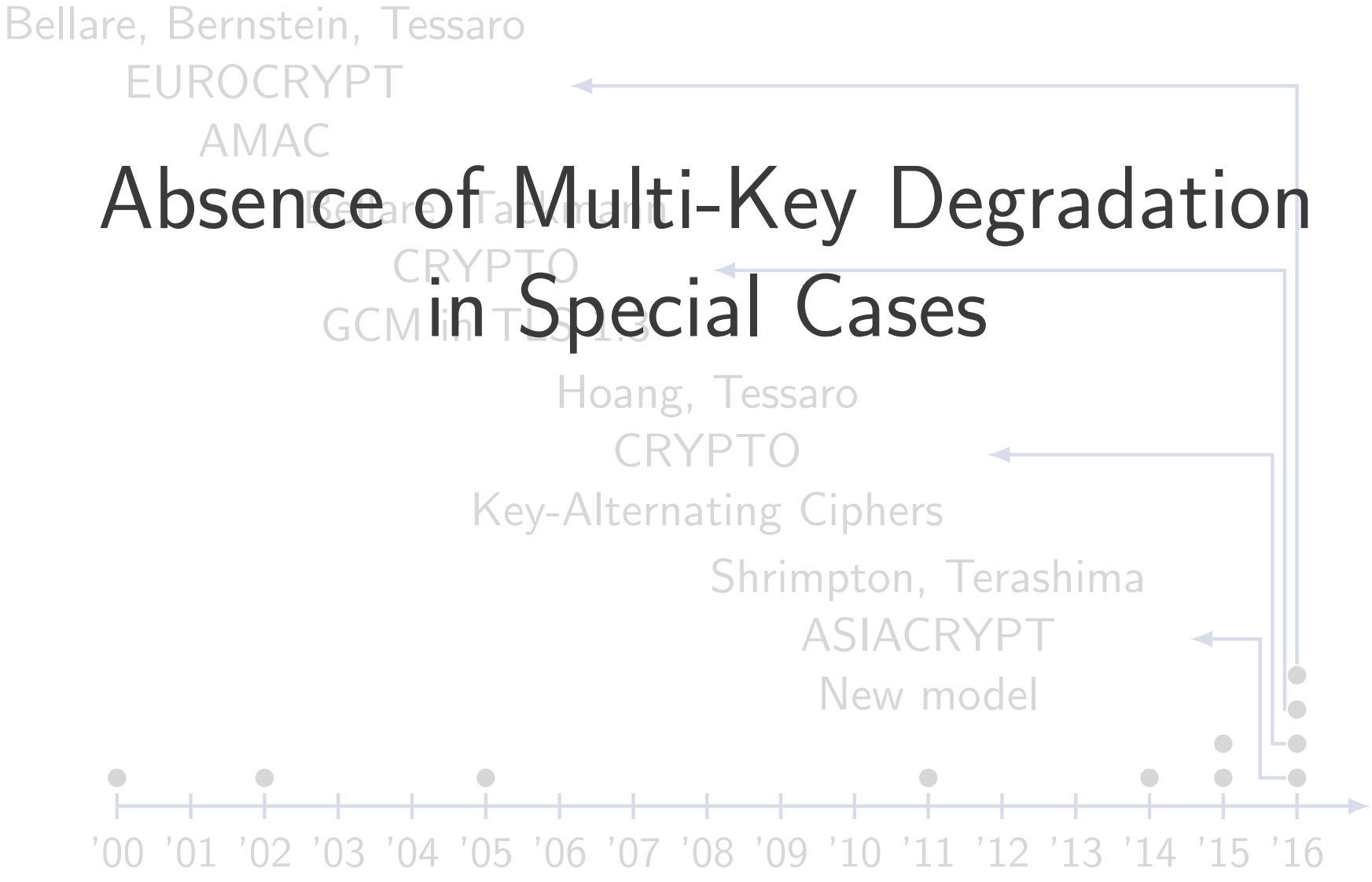
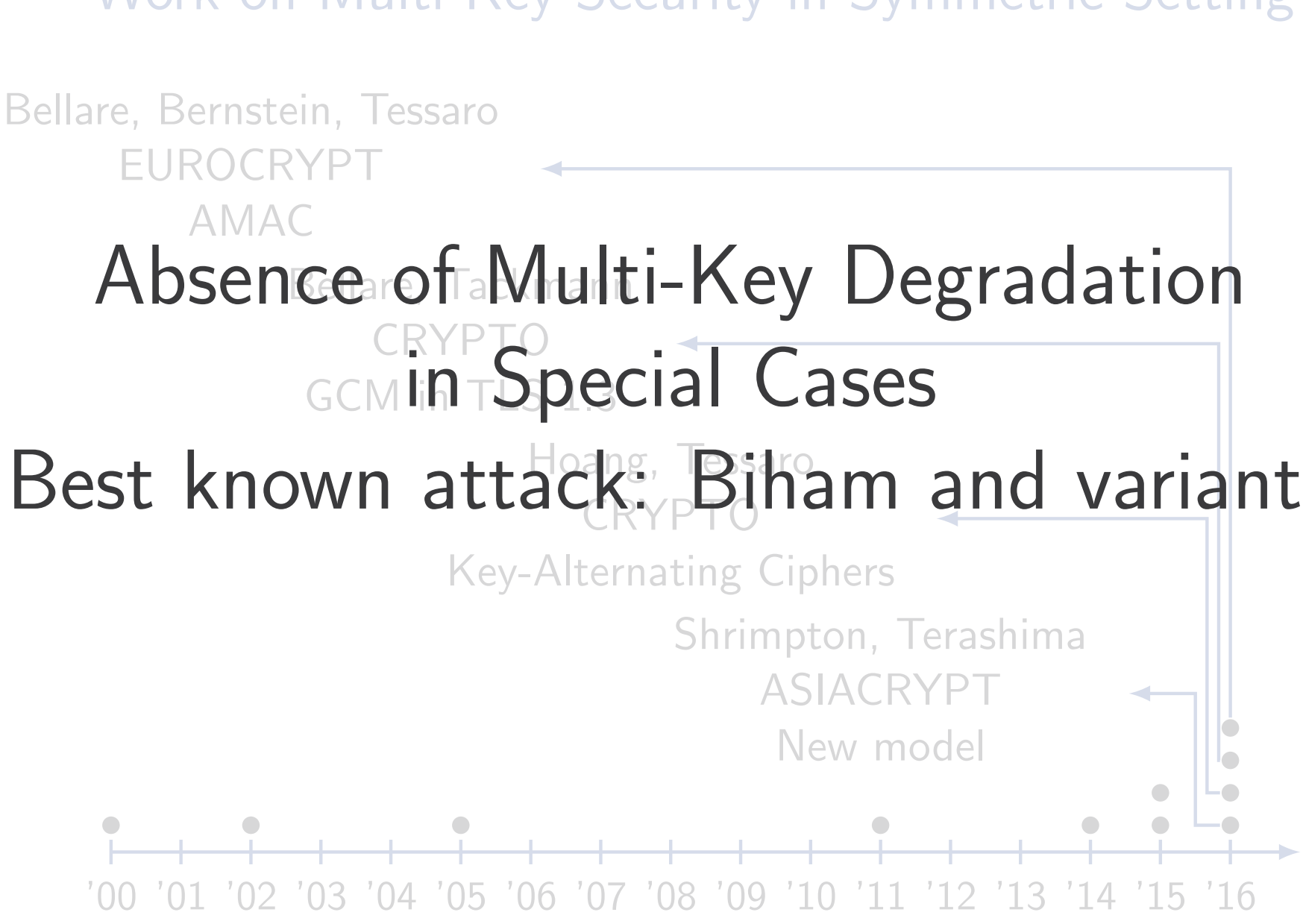




Absence of Multi-Key Degradation in Special Cases

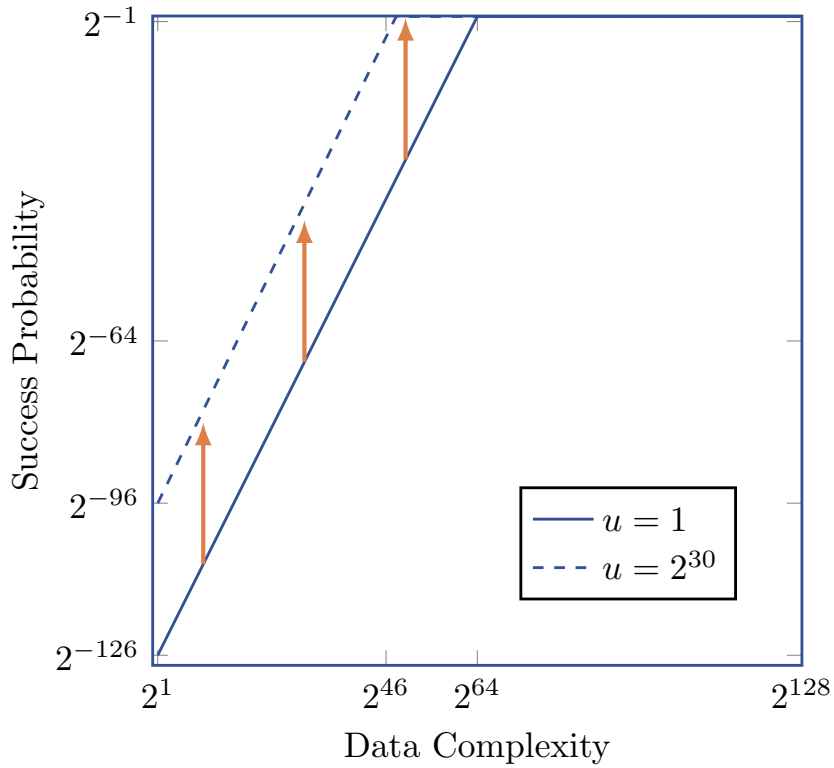


Absence of Multi-Key Degradation in Special Cases

Best known attack: Biham and variants

Our Work

1. Set out to understand gap



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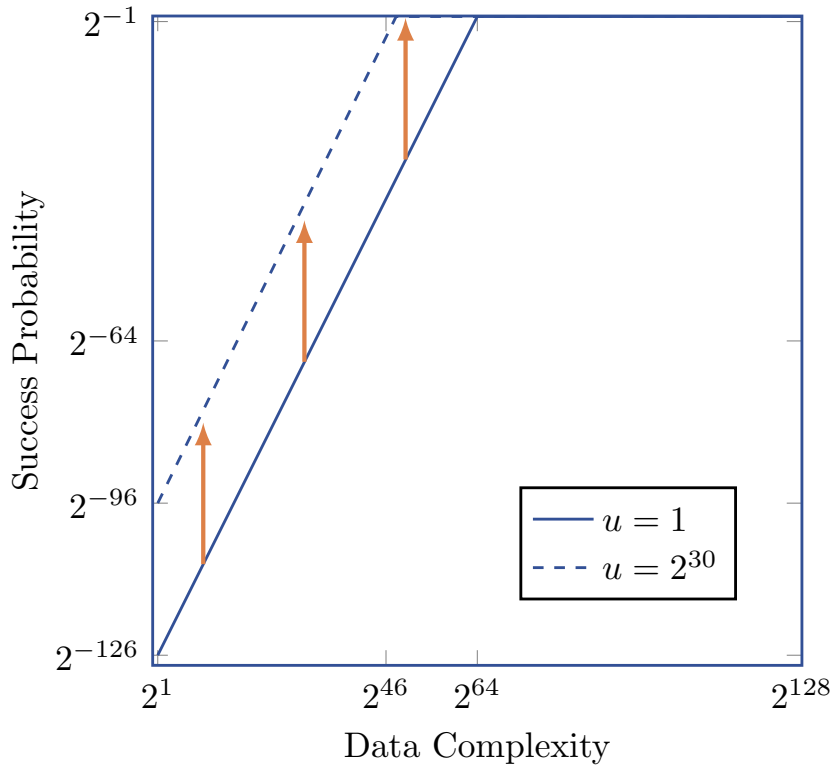
2. Characterization of multi-key setting: necessary and sufficient condition for degradation



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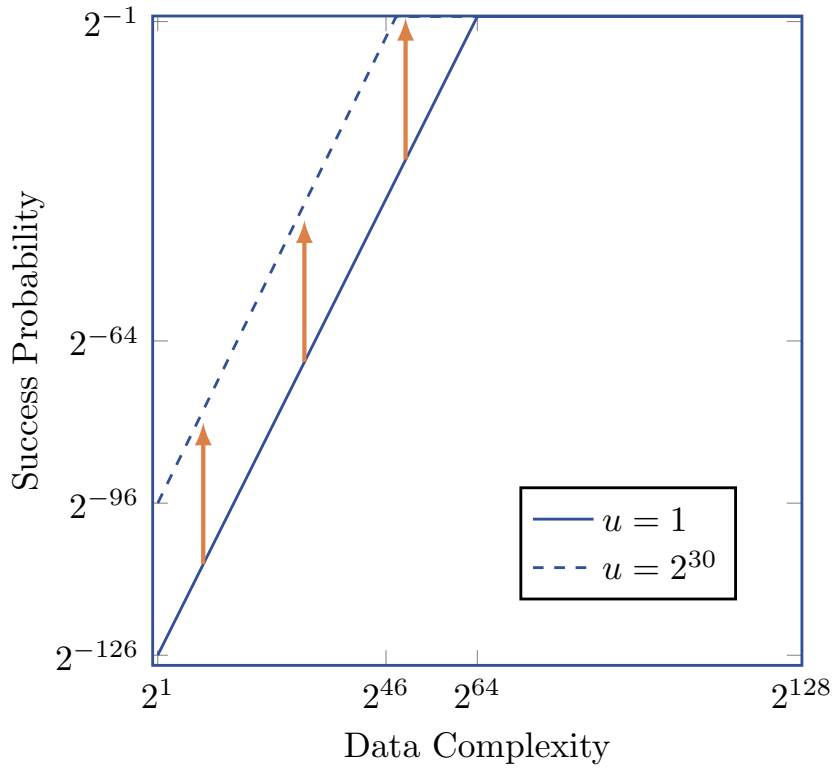
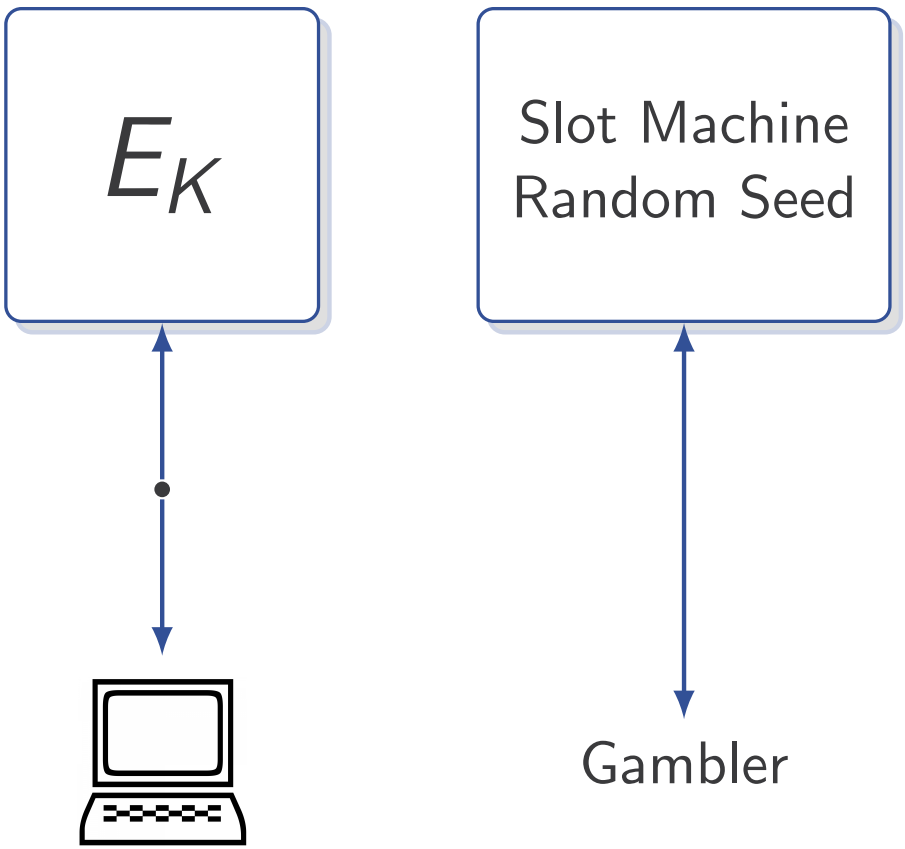
3. Proved in abstract setting, applied to GCM



Matching attacks?

Slot Machine Scenario



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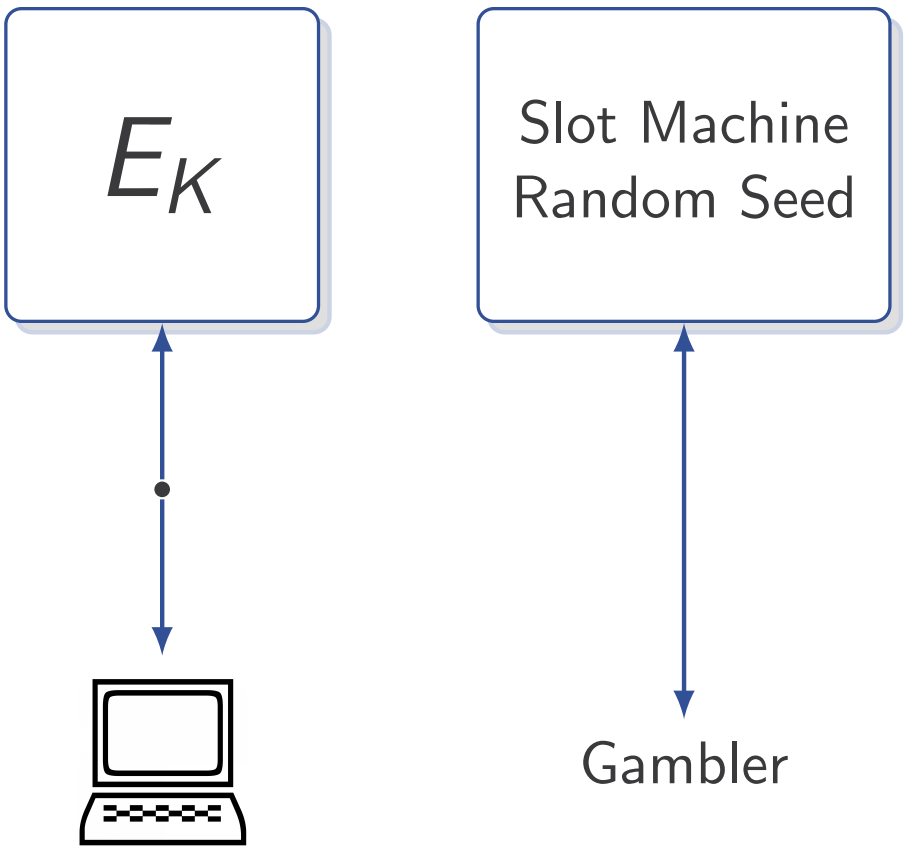

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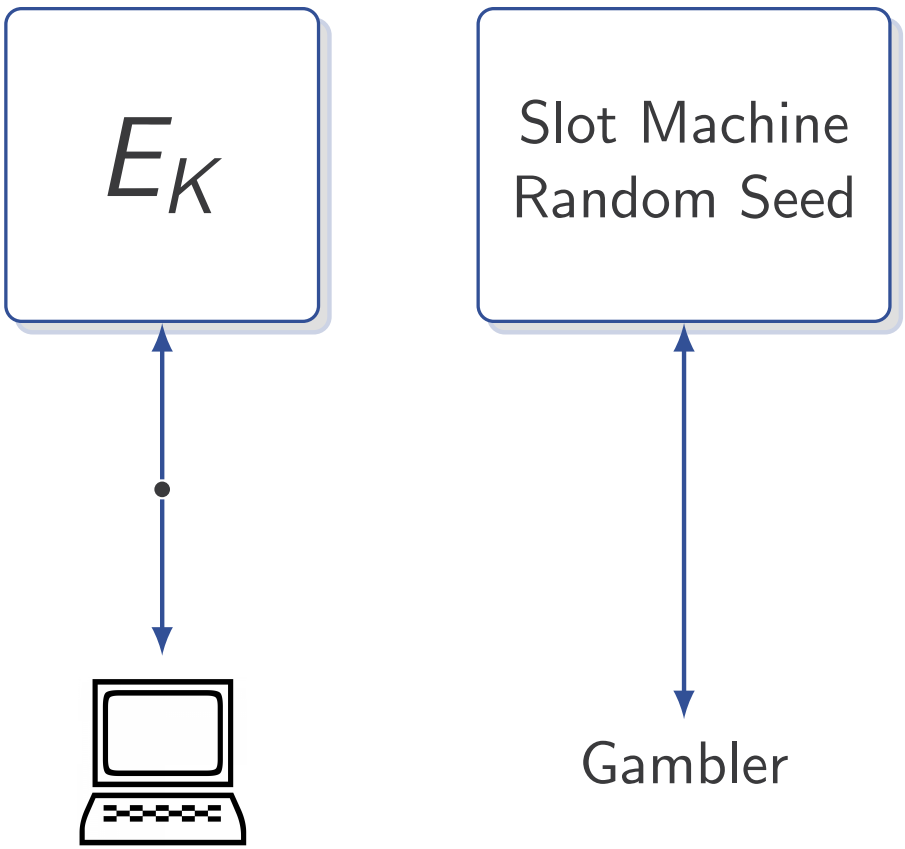

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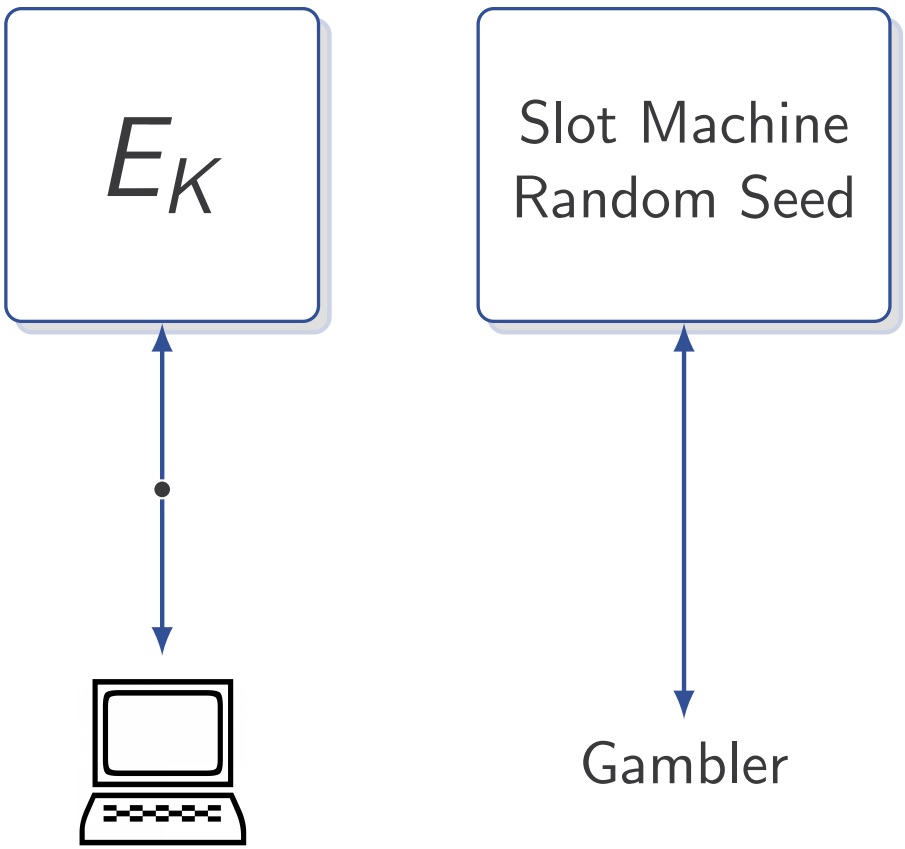

1. Data complexity = money
2. Success = jackpot
3. Single-key setting = access to one slot machine

Slot Machine Scenario

Casino Setting

500 coin budget

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500 coin budget, 100 slot machines

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500 coin budget, 100 slot machines
all 500 coins on 1 machine
vs
500 coins distributed somehow over 100 machines

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Counter-intuitive, ever possible? Yes.

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2. One slot machine: either lucky or not.
3. One hundred slot machines: find lucky machine, focus on that one

Lucky Machines in Cryptography

Weak keys

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Weak-key recovery: computational complexity 2^{16}, data complexity 2.

Table: Midori64 key recovery

	$u = 1$	$u = 2^{16}$
Computational cost	2^{16}	2^{17}
Data cost	2	$u + 2$
Success Estimate	2^{-96}	2^{-80}

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1. Biham key-recovery attack if key size is not too big

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1. Biham key-recovery attack if key size is not too big
2. Exploit Weak instances if present
3. What else can happen?

Necessary and Sufficient Condition

Setting 1
One slot machine
300 coins

Necessary and Sufficient Condition

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Setting 2
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Friend has played on slot machine with 200 coins
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$$\text{Setting 1 Jackpot probability} \leq \text{Setting 2 Jackpot probability}$$

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Lucky Machines Excluded

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Lucky Machines Excluded

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Setting 1: possibility of interacting with lucky machine
$\Rightarrow$ jackpot probability might be higher

Translation to Oracles and Games

Setting 1
An oracle and game
maximum q queries

Setting 2
An oracle and game
maximum q queries
A transcript representing past history

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Setting 1 Adversarial Sucess $\leq$
Setting 2 Adversarial Success given transcript is satisfied

for all transcripts below some cost

then no advantage interacting with multiple oracle instances

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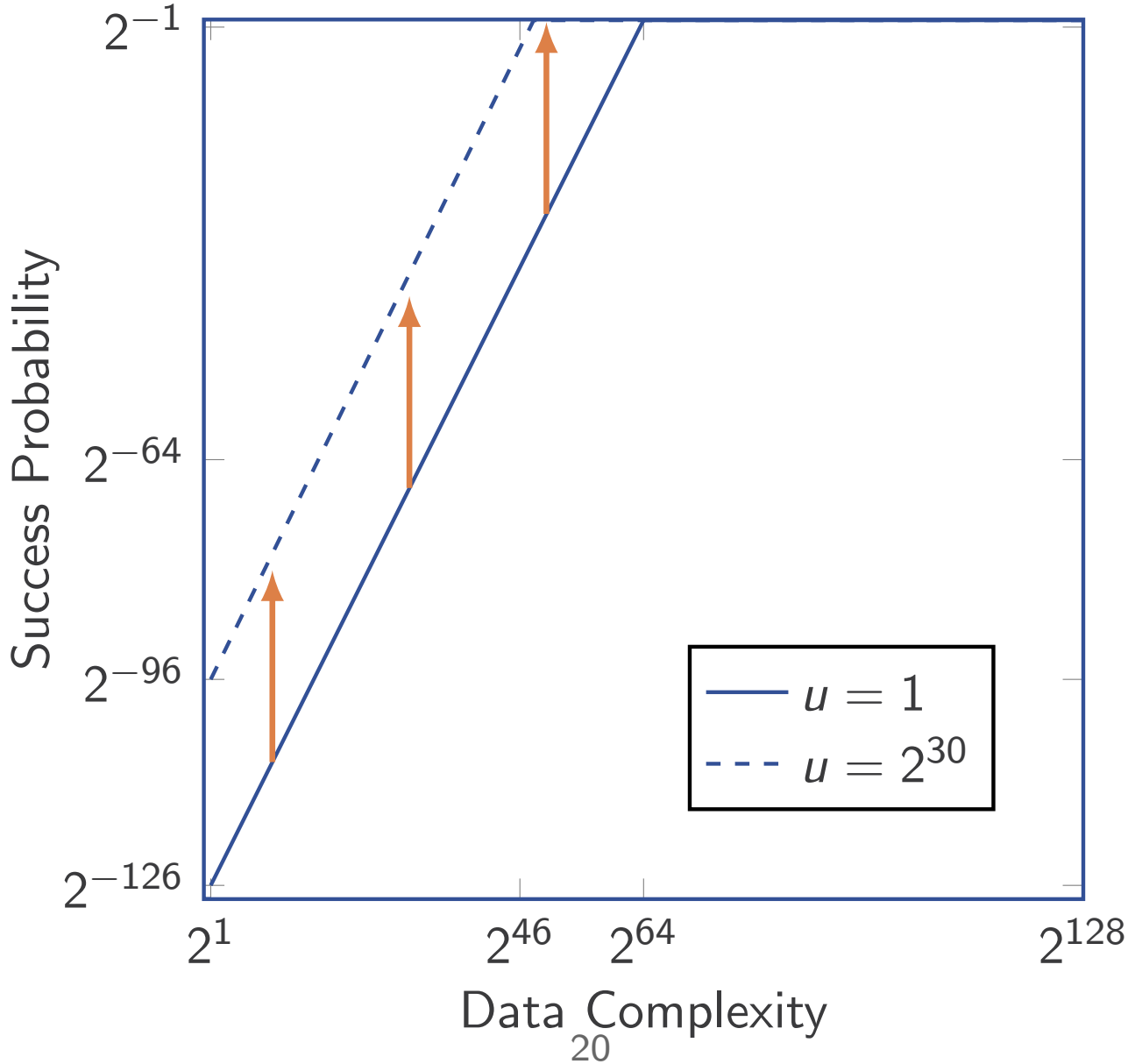
This is only intuition, proper formalization introduces subtleties!
(Information-theoretic setting, adversaries must be optimal, queries are bounded,. . .)

GCM has no Multi-Key Degradation



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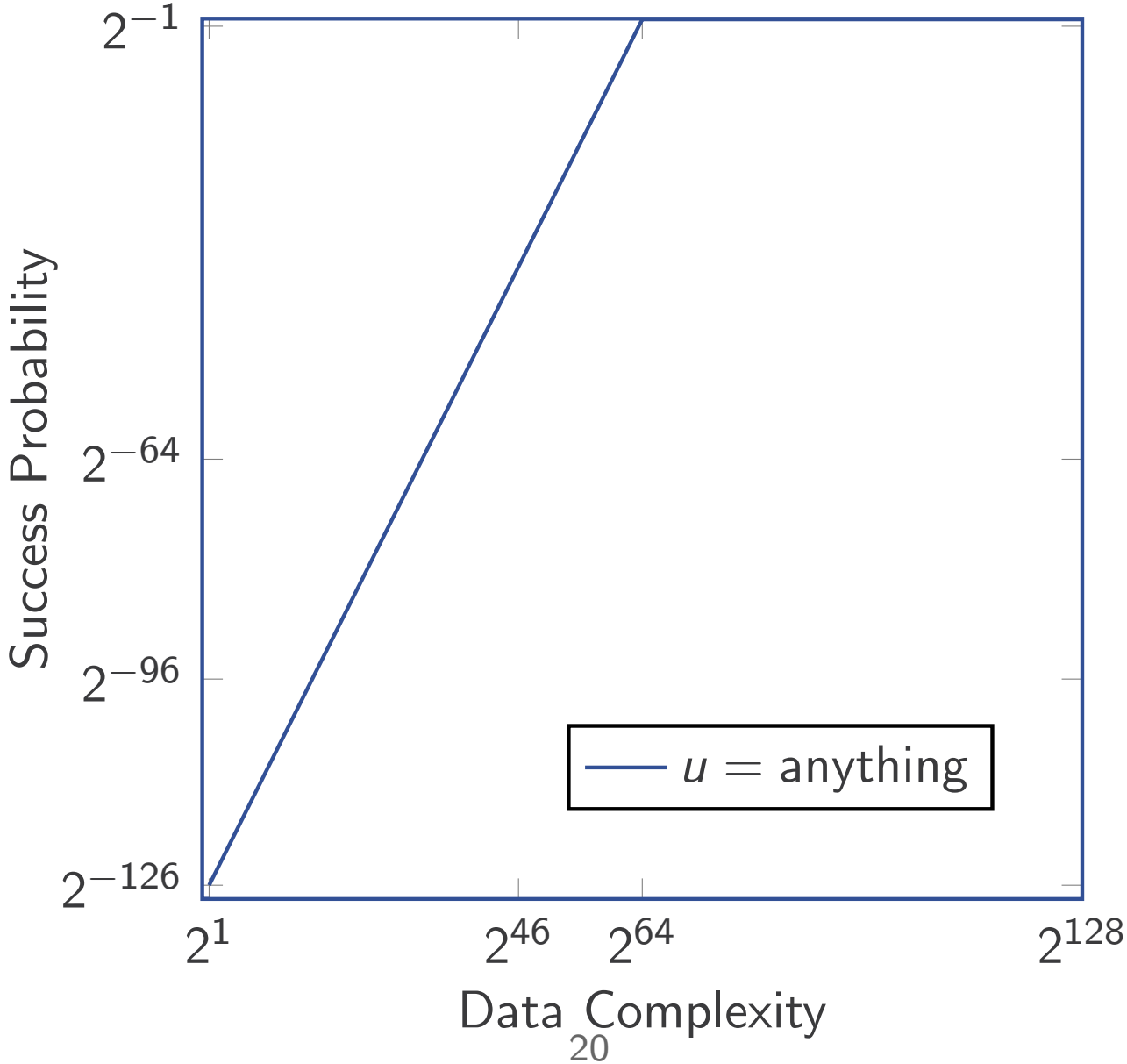

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