

XPX: Generalized Tweakable Even-Mansour with Improved Security Guarantees

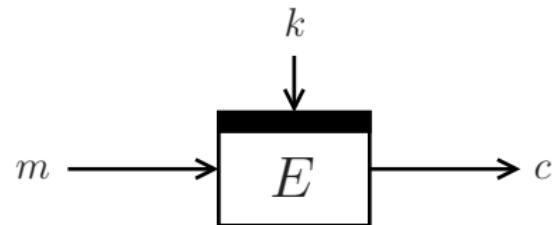
Bart Mennink
KU Leuven (Belgium)

ASK 2015

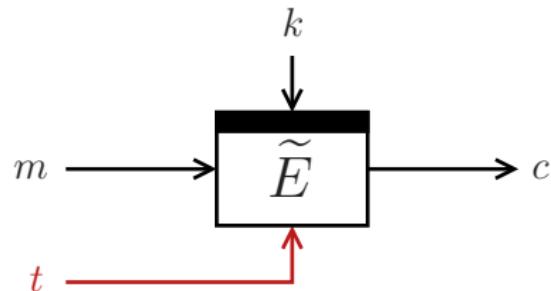
October 2, 2015



Tweakable Blockciphers

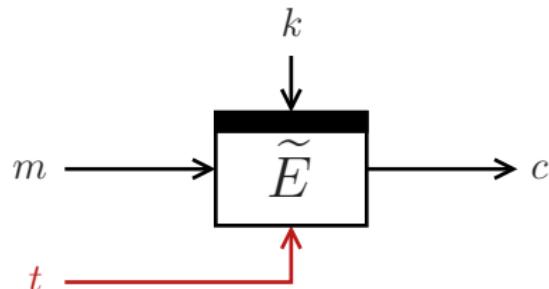


Tweakable Blockciphers



- Tweak: flexibility to the cipher
- Each tweak gives different permutation

Tweakable Blockciphers



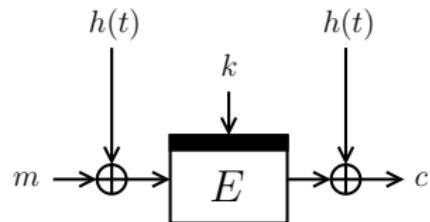
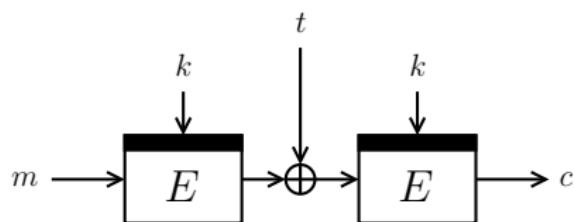
- Tweak: flexibility to the cipher
- Each tweak gives different permutation
- Three approaches:
 - from scratch
 - from blockcipher
 - from permutation

Tweakable Blockciphers from Scratch

- Hasty Pudding Cipher [Sch98]
 - AES submission, “first tweakable cipher”
- Mercy [Cro01]
 - Disk encryption
- Threefish [FLS+07]
 - SHA-3 submission Skein
- TWEAKY [JNP14]
 - CAESAR submissions Deoxys, Joltik, KIASU

Tweakable Blockciphers from Blockcipher

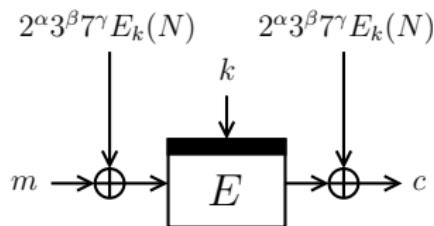
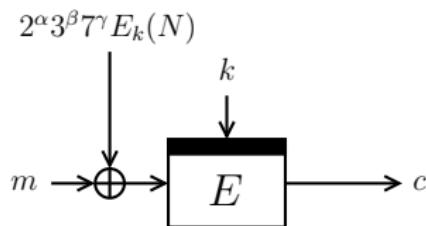
- LRW₁ and LRW₂ by Liskov et al. (2002):



- h is XOR-universal hash

Tweakable Blockciphers from Blockcipher

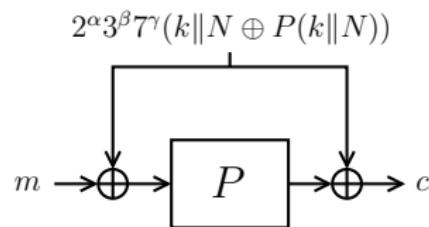
- XE and XEX by Rogaway (2004):



- $(\alpha, \beta, \gamma, N)$ is tweak (simplified)

Tweakable Blockciphers from Permutation

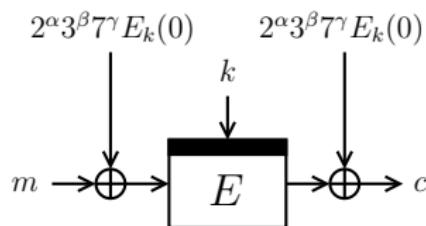
- Minalpher's TEM [STA+14]:



- $(\alpha, \beta, \gamma, N)$ is tweak (simplified)

Tweakable Blockciphers from Permutation

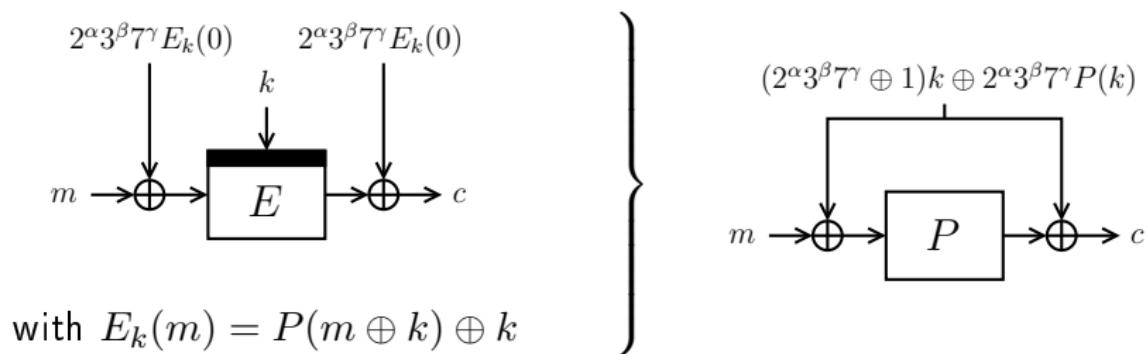
- Prøst [KLL+14] uses XE(X) with Even-Mansour:



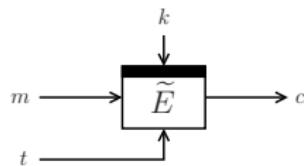
with $E_k(m) = P(m \oplus k) \oplus k$

Tweakable Blockciphers from Permutation

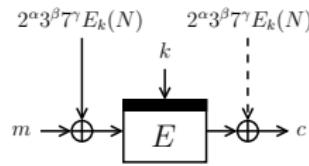
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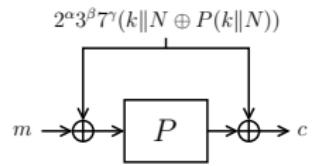
Tweakable Blockciphers in CAESAR



TWEAKY

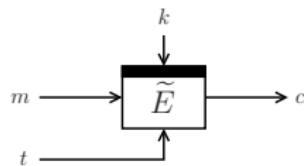


XE/XEX-inspired



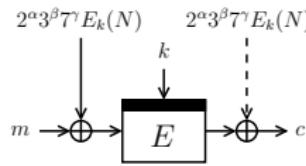
TEM-inspired

Tweakable Blockciphers in CAESAR



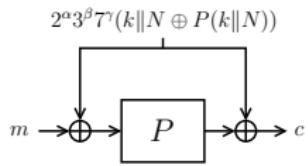
TWEAKY

Deoxys,
Joltik,
KIASU



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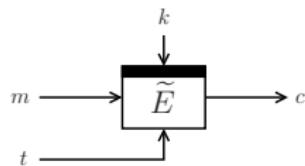
AEZ, CBA, COBRA,
COPA, ELMdD, iFeed,
Marble, OCB, OMD,
OTR, POET, SHELL



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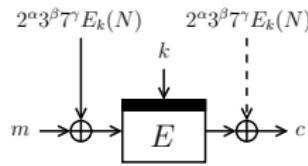
Minalpher,
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Tweakable Blockciphers in CAESAR



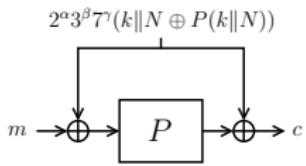
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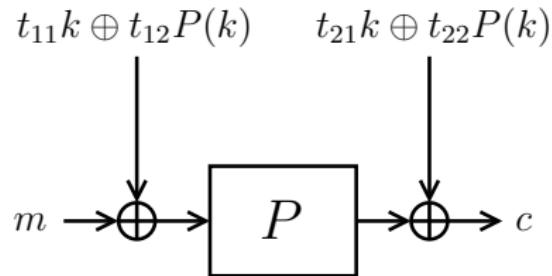
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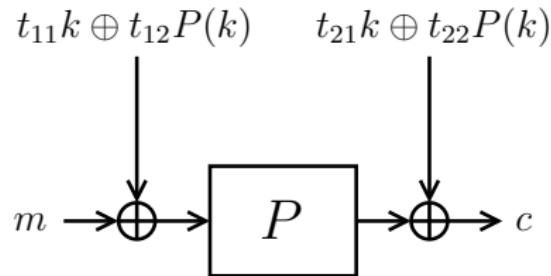
Minalpher,
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We generalize this



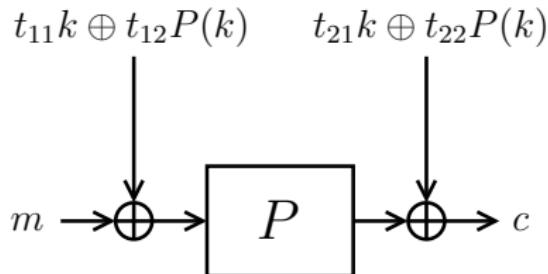
Tweak Set

- $(t_{11}, t_{12}, t_{21}, t_{22})$ from some tweak set $\mathcal{T} \subseteq (\{0, 1\}^n)^4$
- \mathcal{T} can (still) be any set



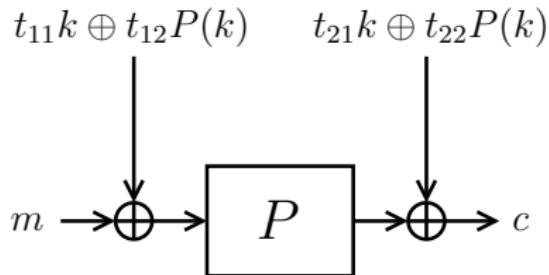
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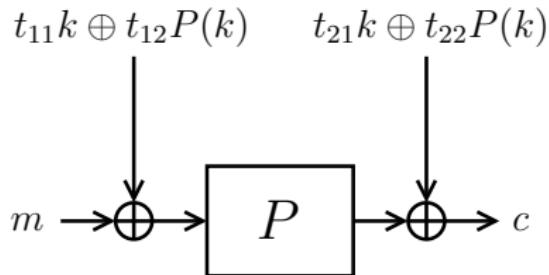
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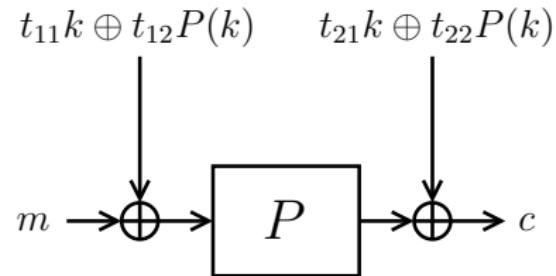
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 - ② “Normal” \mathcal{T} \longrightarrow single-key secure



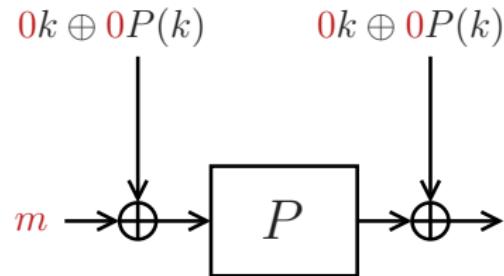
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 - ① “Stupid” \mathcal{T} \longrightarrow insecure
 - ② “Normal” \mathcal{T} \longrightarrow single-key secure
 - ③ “Strong” \mathcal{T} \longrightarrow related-key secure

XPX: Valid Tweaks

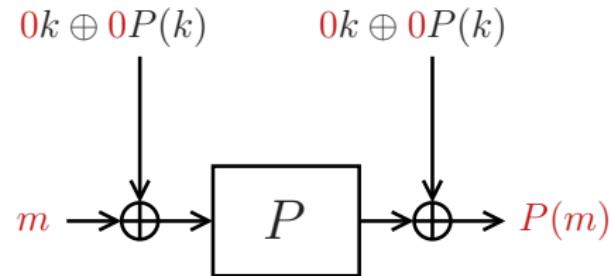


XPX: Valid Tweaks



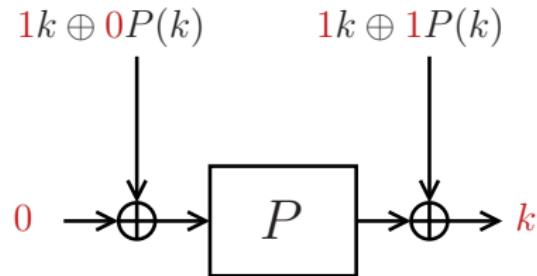
$$(0, 0, 0, 0) \in \mathcal{T}$$

XPX: Valid Tweaks



$$(0, 0, 0, 0) \in \mathcal{T} \implies \text{XPX}_k((0, 0, 0, 0), m) = P(m)$$

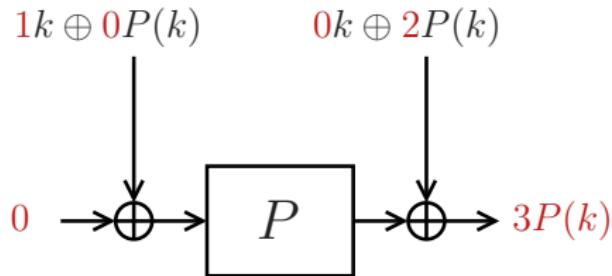
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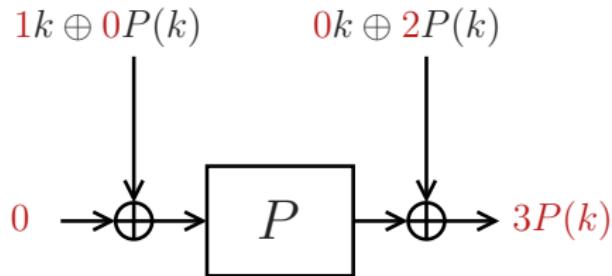


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$$(1, 0, 0, 2) \in \mathcal{T} \implies \text{XPX}_k((1, 0, 0, 2), 0) = 3P(k)$$

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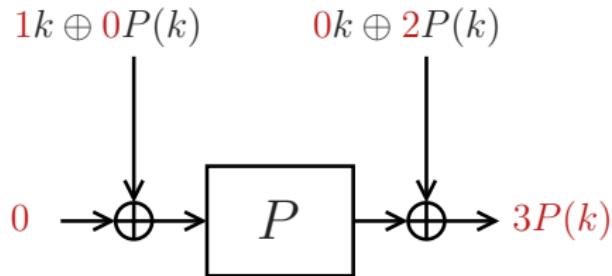
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...

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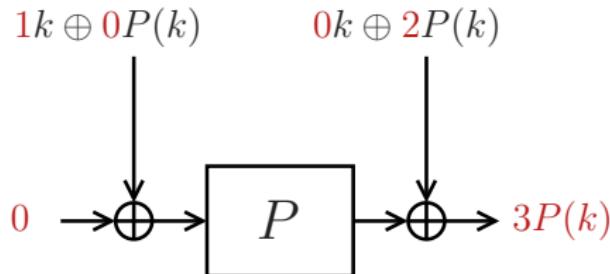
...

...

“Valid” Tweak Sets

- Technical definition to eliminate trivial cases

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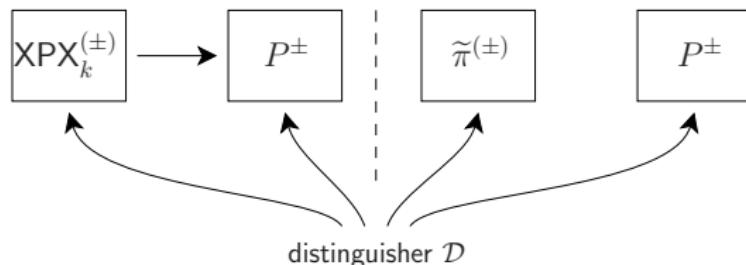
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“Valid” Tweak Sets

- Technical definition to eliminate trivial cases
- Proven to be minimal: \mathcal{T} invalid \Rightarrow XPX insecure

XPX: Single-Key Security

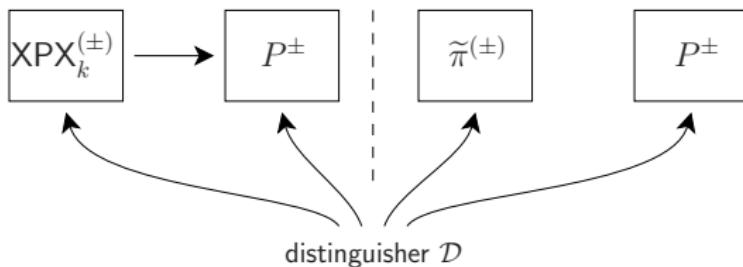
(Strong) Tweakable PRP



- Information-theoretic indistinguishability
 - $\tilde{\pi}$ ideal tweakable permutation
 - P ideal permutation
 - k secret key

XPX: Single-Key Security

(Strong) Tweakable PRP

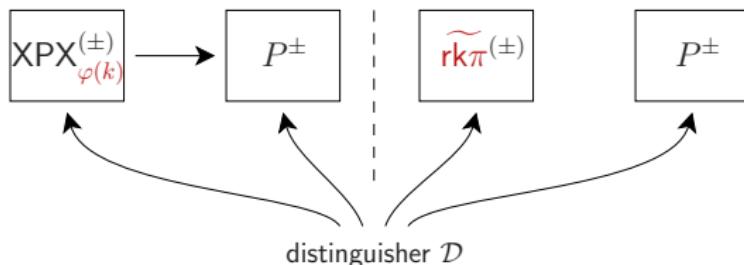


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$$\mathcal{T} \text{ is valid} \implies \text{XPX is (S)TPRP up to } \mathcal{O}\left(\frac{q^2 + qr}{2^n}\right)$$

XPX: Related-Key Security

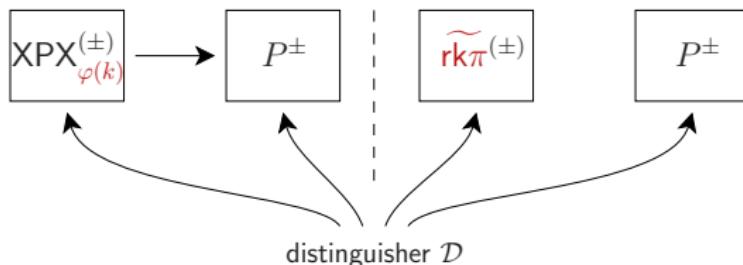
Related-Key (Strong) Tweakable PRP



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XPX: Related-Key Security

Related-Key (Strong) Tweakable PRP



- Information-theoretic indistinguishability
 - $\widetilde{rk}\pi$ ideal tweakable related-key permutation
 - P ideal permutation
 - k secret key
- \mathcal{D} restricted to some set of key-deriving functions Φ

XPX: Related-Key Security

Key-Deriving Functions (Informal)

- Φ_{\oplus} : all functions $k \mapsto k \oplus \delta$

XPX: Related-Key Security

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- Note: maskings in XPX are $t_{i1}k \oplus t_{i2}P(k)$

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Results

if \mathcal{T} is valid, and for all tweaks:	security	Φ
$t_{12} \neq 0$	TPRP	Φ_{\oplus}
$t_{12}, t_{22} \neq 0$ and $(t_{21}, t_{22}) \neq (0, 1)$	STPRP	Φ_{\oplus}

XPX: Related-Key Security

Key-Deriving Functions (Informal)

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$t_{12} \neq 0$	TPRP	Φ_{\oplus}
$t_{12}, t_{22} \neq 0$ and $(t_{21}, t_{22}) \neq (0, 1)$	STPRP	Φ_{\oplus}
$t_{11}, t_{12} \neq 0$	TPRP	$\Phi_{P\oplus}$
$t_{11}, t_{12}, t_{21}, t_{22} \neq 0$	STPRP	$\Phi_{P\oplus}$

XPX: Security Proof Techniques

Patarin's H-coefficient Technique

- Each conversation defines a transcript
- Define **good** and **bad** transcripts

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$$\mathbf{Adv}_{\text{XPX}}^{\text{rk-}(s)\text{prp}}(\mathcal{D}) \leq \varepsilon + \Pr \left[\text{bad transcript for } (\widetilde{\text{rk}\pi}, P) \right]$$

↑— prob. ratio for **good** transcripts

XPK: Security Proof Techniques

Patarin's H-coefficient Technique

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↑— prob. ratio for **good** transcripts

- Trade-off: define **bad** transcripts smartly!

XPX: Security Proof Techniques

Before the Interaction

- Reveal dedicated construction queries

After the Interaction

- Reveal key information
 - Single-key: k and $P(k)$
 - Φ_{\oplus} -related-key: k and $P(k \oplus \delta)$
 - $\Phi_{P\oplus}$ -related-key: k and $P(k \oplus \delta)$ and $P^{-1}(P(k) \oplus \varepsilon)$

Bounding the Advantage

- Smart definition of **bad** transcripts

XPX Covers Even-Mansour



for $\mathcal{T} = \{(1, 0, 1, 0)\}$

XPX Covers Even-Mansour



for $\mathcal{T} = \{(1, 0, 1, 0)\}$

- Single-key STPRP secure (surprise?)

XPX Covers Even-Mansour



for $\mathcal{T} = \{(1, 0, 1, 0)\}$

- Single-key STPRP secure (surprise?)
- Generally, if $|\mathcal{T}| = 1$, XPX is a normal blockcipher

XPX Covers XEX With Even-Mansour



$$\text{for } \mathcal{T} = \left\{ \begin{array}{l} (2^\alpha 3^\beta 7^\gamma \oplus 1, 2^\alpha 3^\beta 7^\gamma, \\ 2^\alpha 3^\beta 7^\gamma \oplus 1, 2^\alpha 3^\beta 7^\gamma) \end{array} \mid (\alpha, \beta, \gamma) \in \{\text{XEX-tweaks}\} \right\}$$

- (α, β, γ) is in fact the “real” tweak

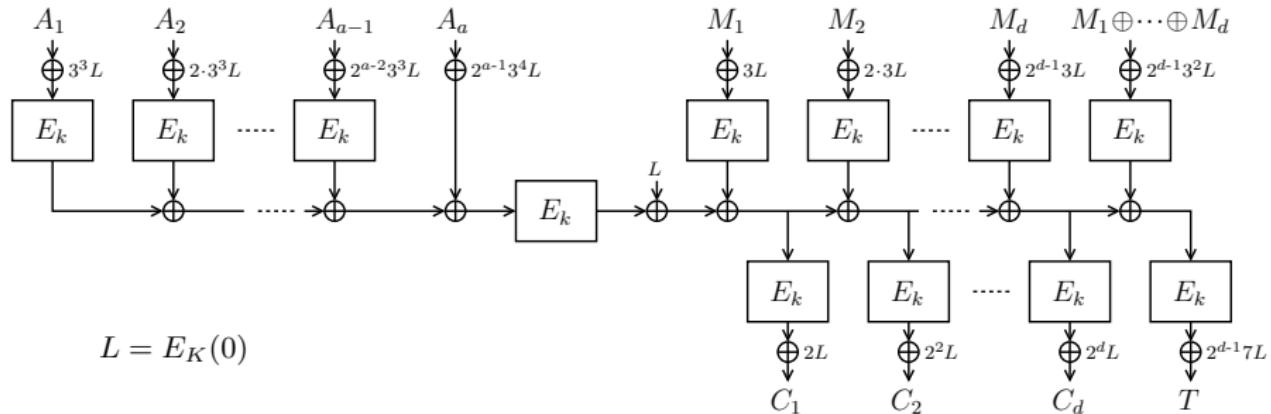
XEX Covers XEX With Even-Mansour



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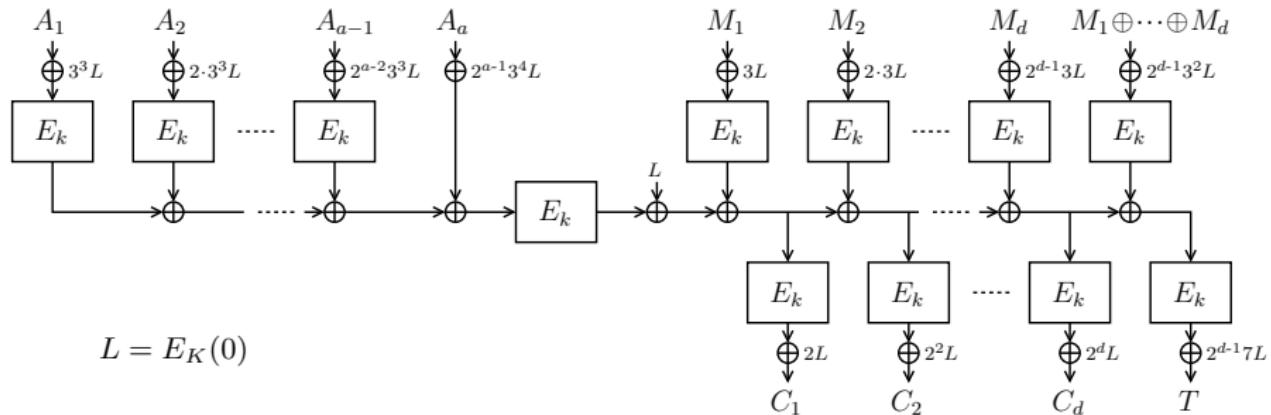
- (α, β, γ) is in fact the “real” tweak
- $\Phi_{P \oplus}$ -related-key STPRP secure (if $2^{\alpha}3^{\beta}7^{\gamma} \neq 1$)

Application to AE: COPA



- By Andreeva et al. (2014)
- Implicitly based on XEX based on AES

Application to AE: COPA



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- Implicitly based on XEX based on AES
- Prøst-COPA by Kavun et al. (2014):
COPA based on XEX based on Even-Mansour

Application to AE: COPA

Single-Key Security of COPA



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Single-Key Security of COPA



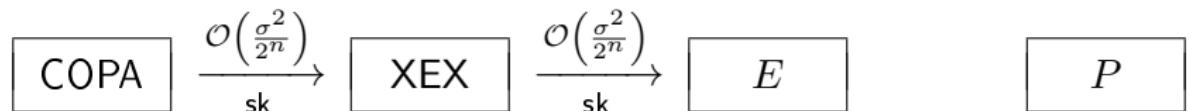
Related-Key Security of COPA

- Approach generalizes for any Φ (proof in paper)



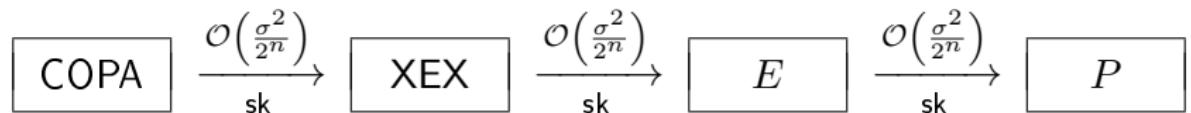
Application to AE: Prøst-COPA

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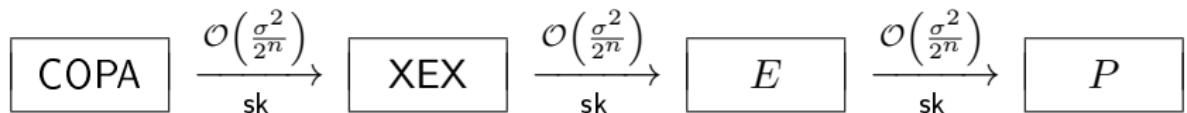
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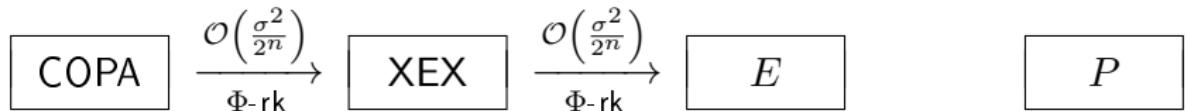


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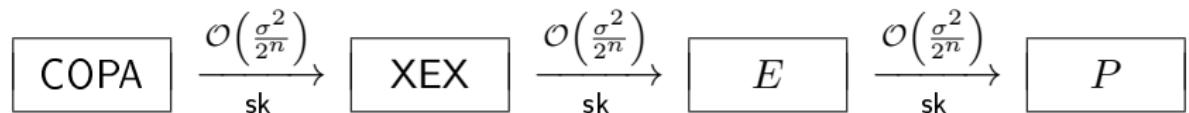


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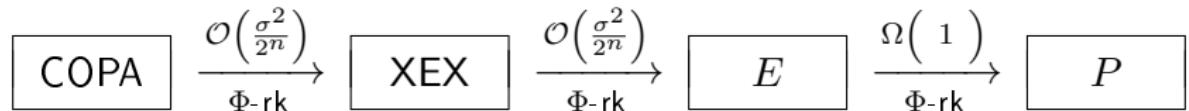


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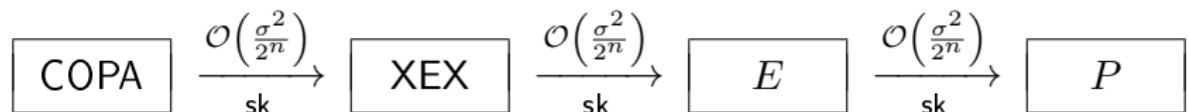


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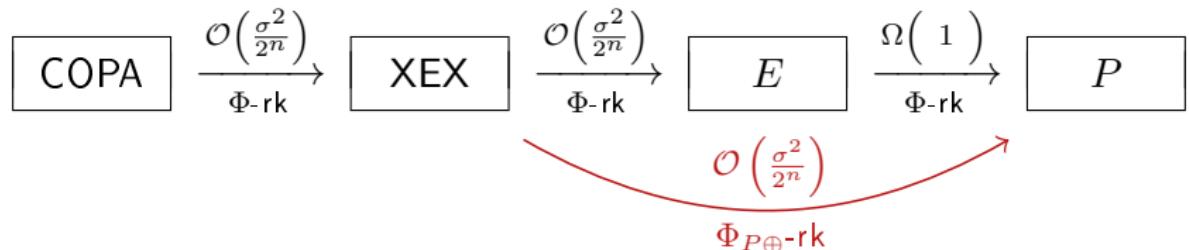


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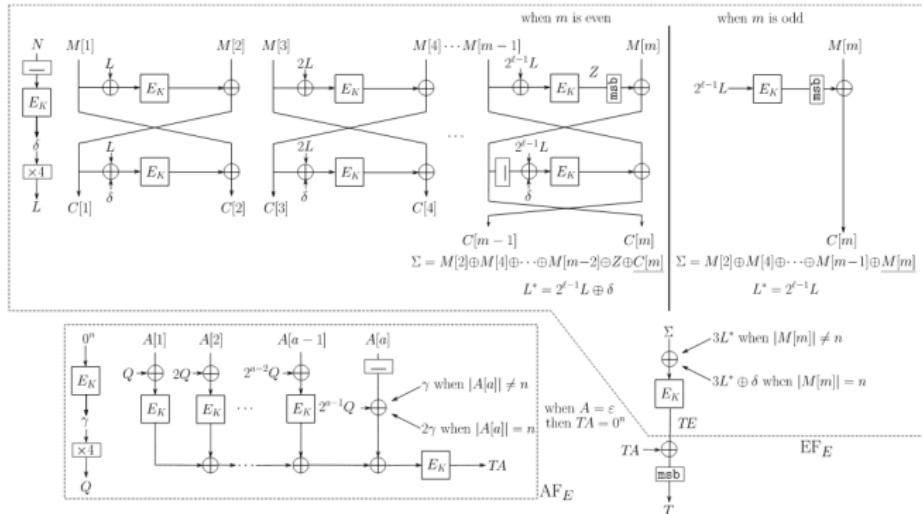
Single-Key Security of Prøst-COPA



Related-Key Security of Prøst-COPA

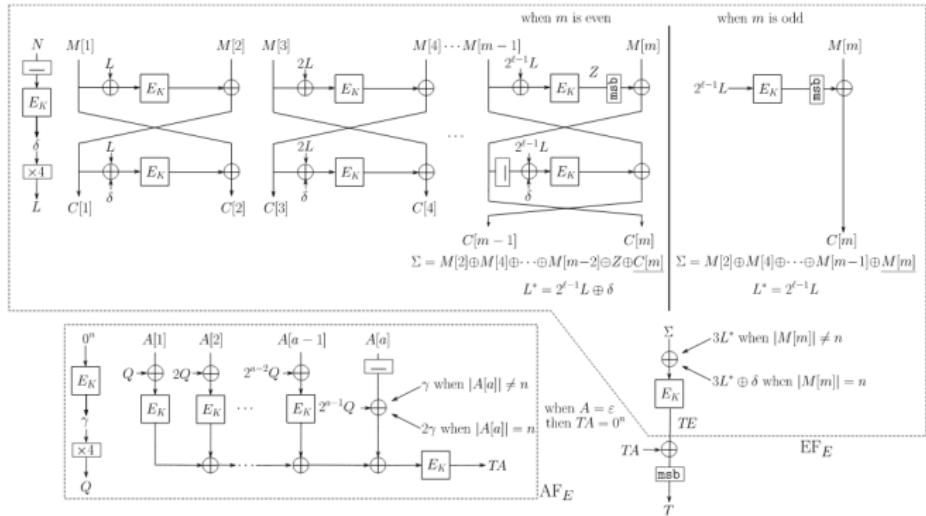


Application to AE: Prøst-OTR



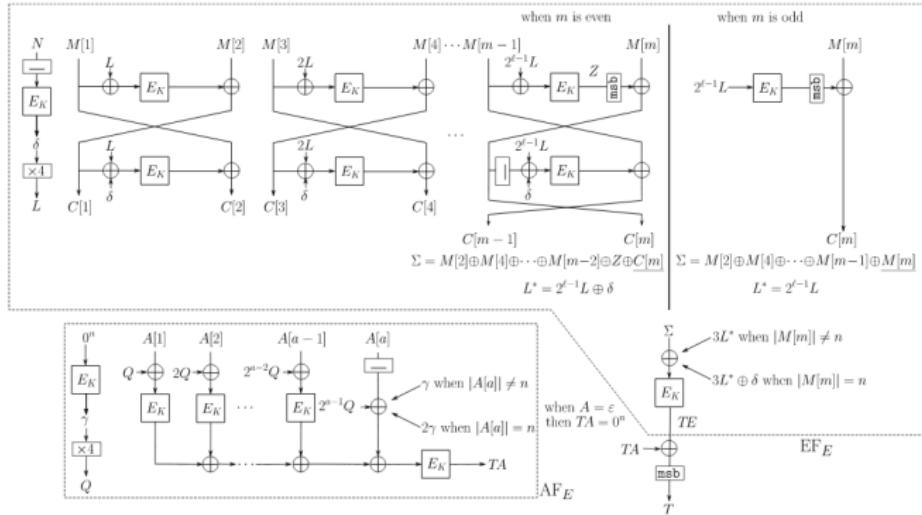
- Prøst-OTR by Kavun et al. (2014):
OTR based on XE based on Even-Mansour

Application to AE: Prøst-OTR



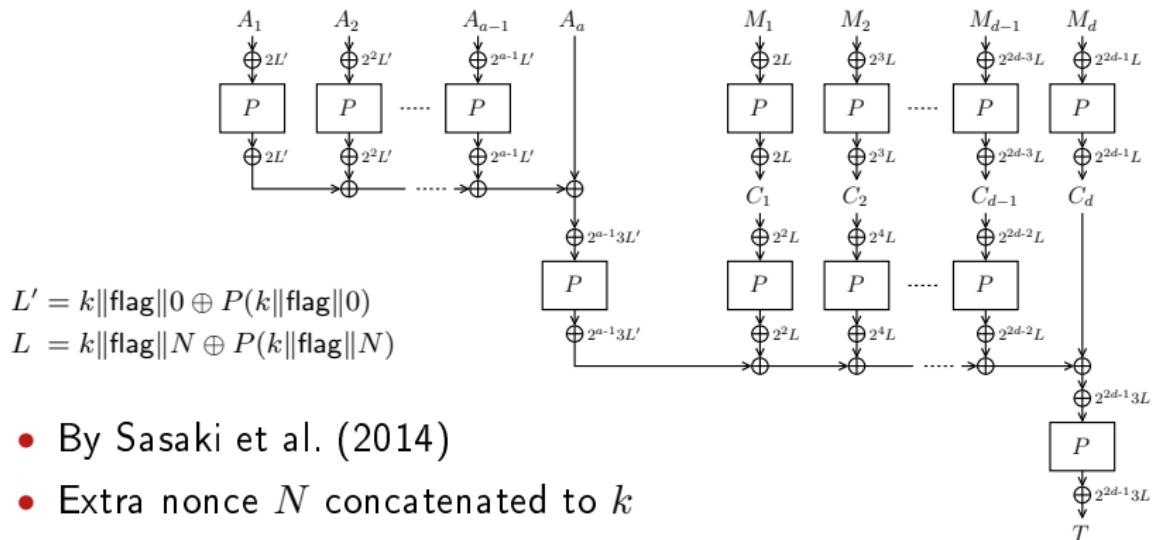
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Application to AE: Prøst-OTR



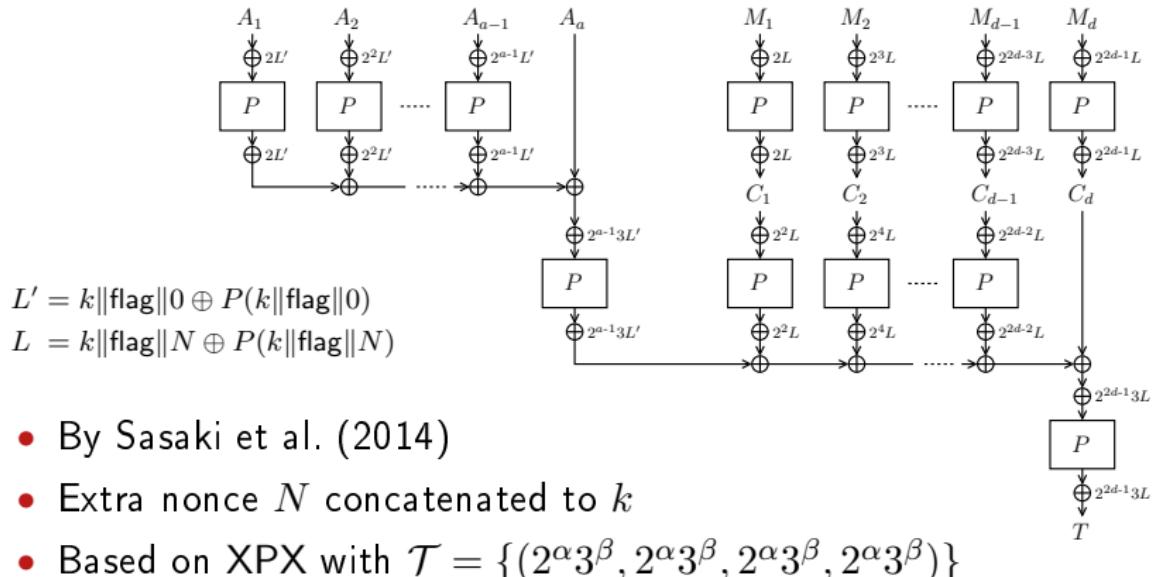
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- Nonce-based masking: $t_{11}k \oplus t_{11}P(k \oplus N)$

Application to AE: Minalpher



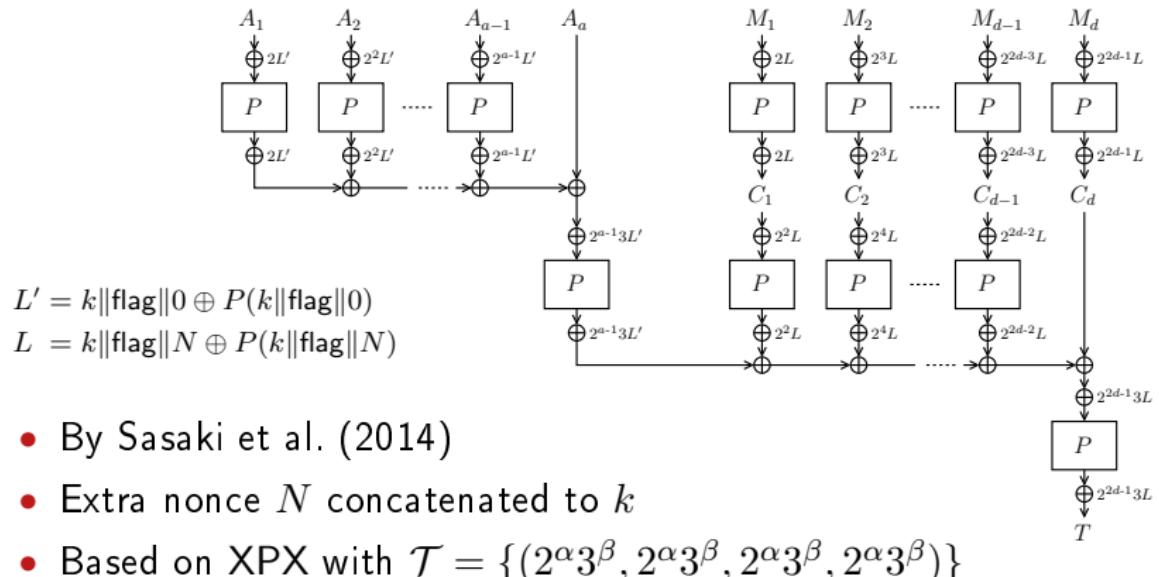
- By Sasaki et al. (2014)
- Extra nonce N concatenated to k

Application to AE: Minalpher



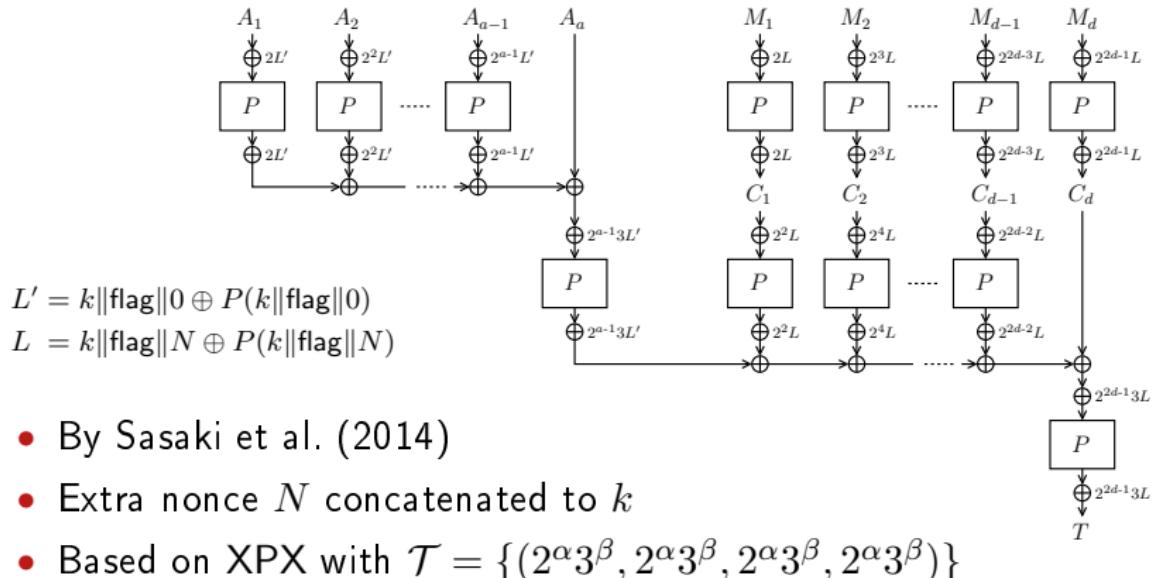
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- Based on XPX with $\mathcal{T} = \{(2^\alpha 3^\beta, 2^\alpha 3^\beta, 2^\alpha 3^\beta, 2^\alpha 3^\beta)\}$

Application to AE: Minalpher



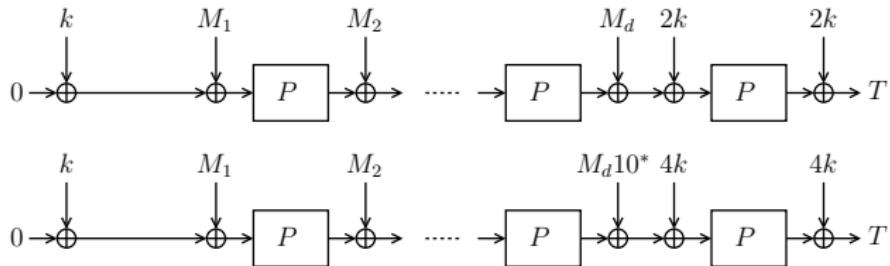
$$\boxed{\text{Minalph.}} \xrightarrow[\Phi\text{-rk}]{\mathcal{O}\left(\frac{\sigma^2}{2^n}\right)} \boxed{\text{XPX}} \quad \boxed{P}$$

Application to AE: Minalpher



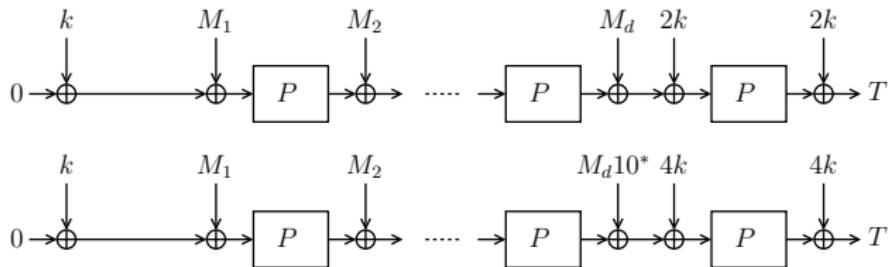
$$\begin{array}{c}
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 \end{array}$$

Application to MAC: Chaskey



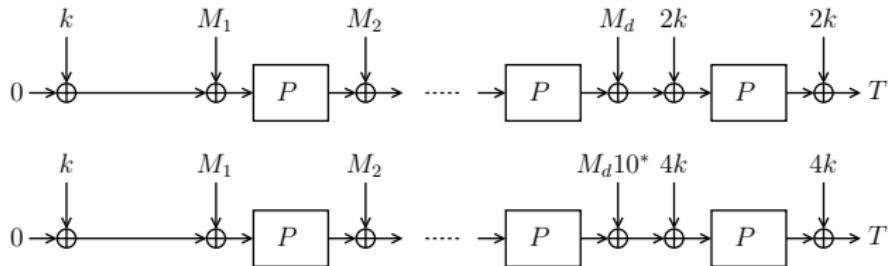
- By Mouha et al. (2014)
- Original proof based on 3 EM's:
$$\begin{cases} E_k(m) = P(m \oplus k) \oplus k \\ E_k(m) = P(m \oplus 3k) \oplus 2k \\ E_k(m) = P(m \oplus 5k) \oplus 4k \end{cases}$$

Application to MAC: Chaskey

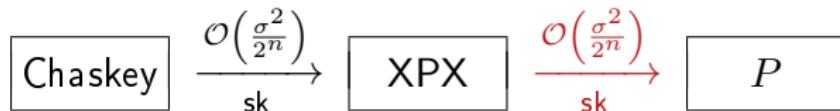


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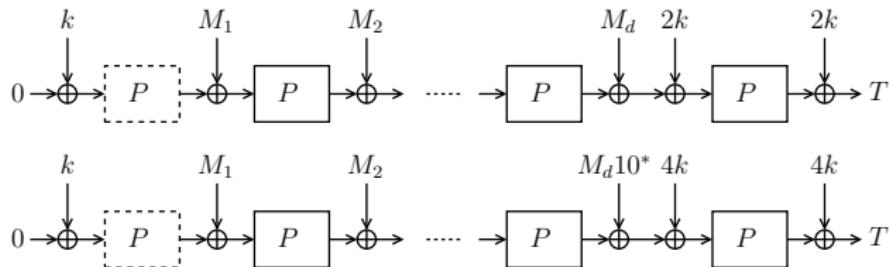
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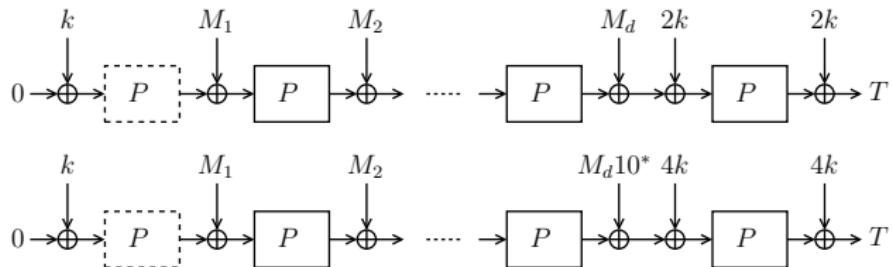


Application to MAC: Adjusted Chaskey



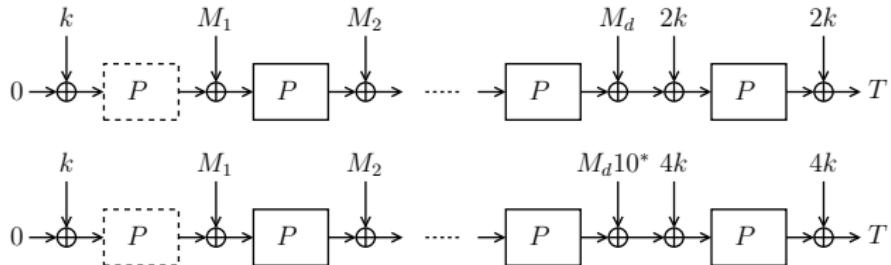
- Extra P -call

Application to MAC: Adjusted Chaskey

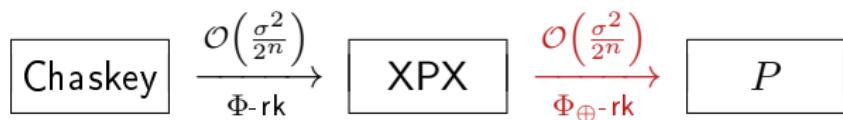


- Extra P -call
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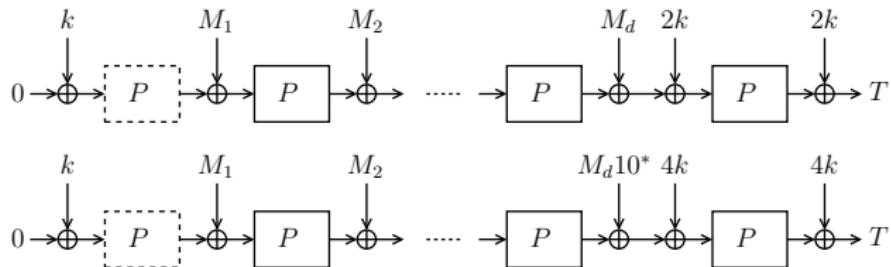
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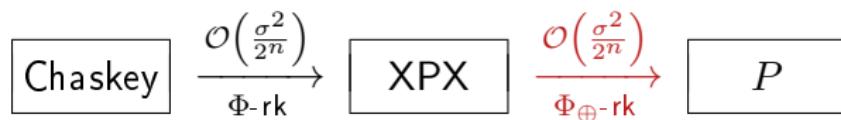
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Application to MAC: Adjusted Chaskey



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- Approach also applies to Keyed Sponges

Security Beyond Birthday Bound?

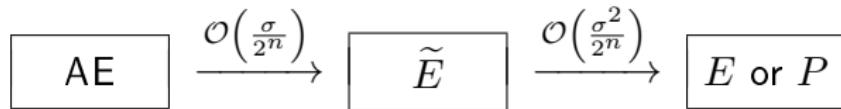
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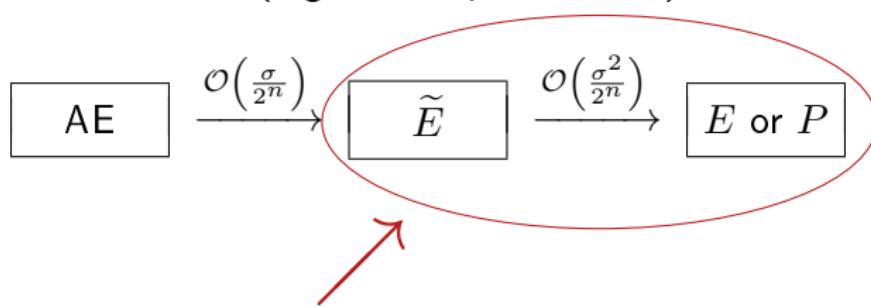
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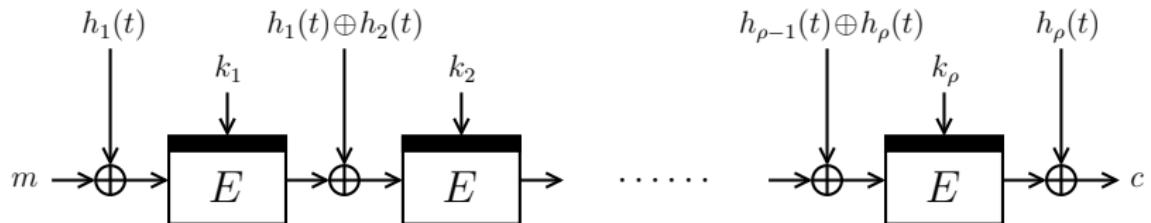
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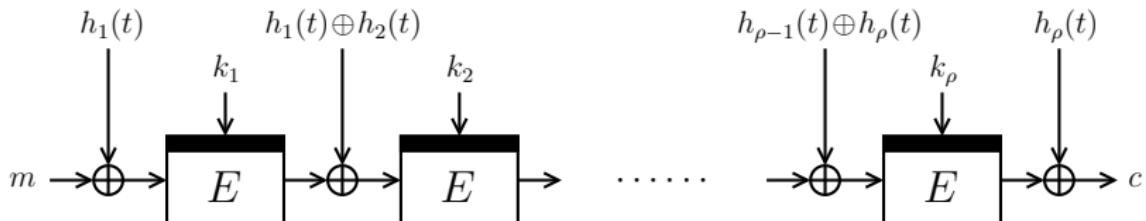
Can we improve this?

BBB Tweakable Blockciphers from Blockcipher



- LRW₂[ρ]: concatenation of ρ LRW₂'s
- k_1, \dots, k_{ρ} and h_1, \dots, h_{ρ} independent

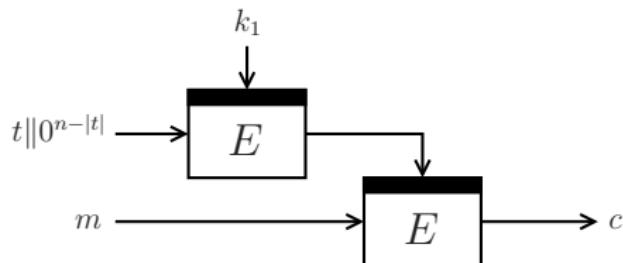
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- k_1, \dots, k_ρ and h_1, \dots, h_ρ independent
- $\rho = 2$: secure up to $2^{2n/3}$ queries [LST12, Pro14]
- $\rho \geq 2$ even: secure up to $2^{\rho n / (\rho + 2)}$ queries [LS13]

BBB Tweakable Blockciphers from Blockcipher

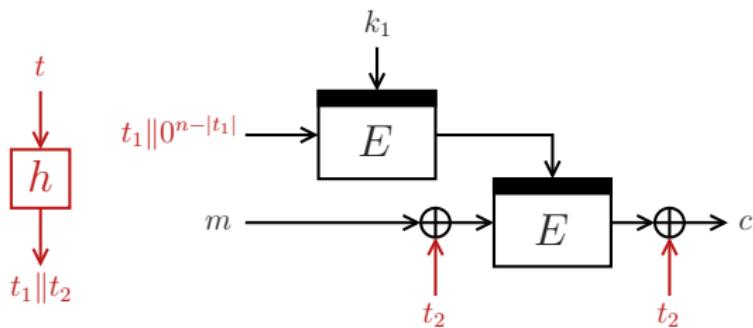
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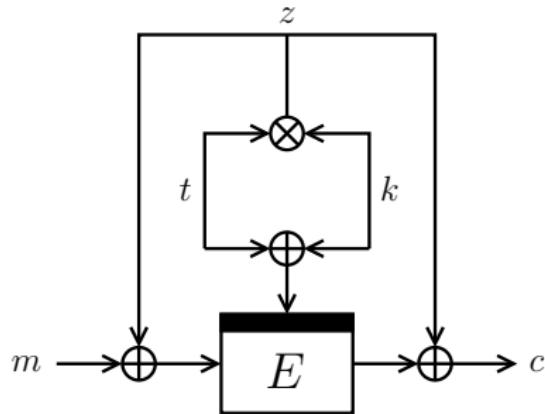
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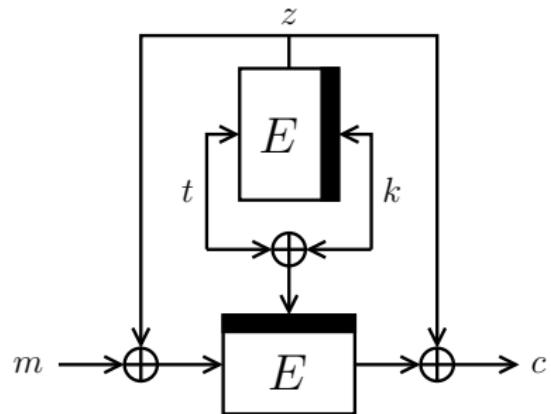
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- Tweak-length extension possible by recent XTX [MI09]

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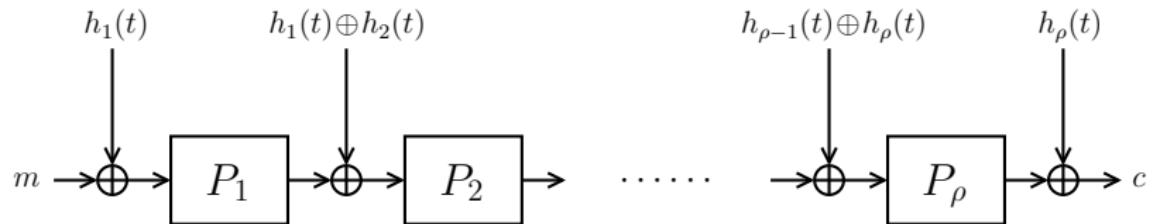


Secure up to $2^{2n/3}$ queries
(one \otimes , one E)



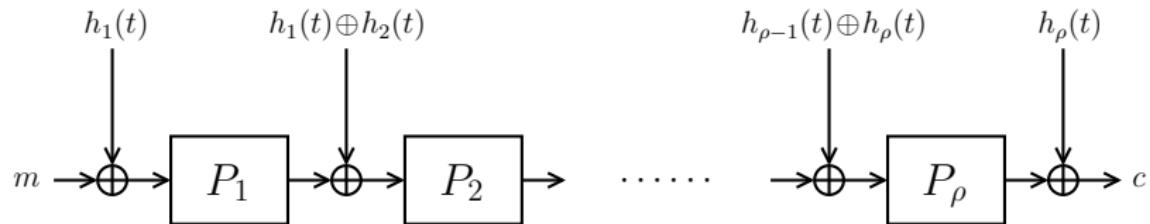
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Conclusions

XPX

- Generalized tweakable Even-Mansour
- Various levels of security
 - Single-key to related-key
- Many applications to AE and MAC

Optimal Secure AE?

- AE with cascaded LRW₂ or TEM: $2^{\rho n}/(\rho+2)$ security,
but using ρ calls to E/P
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Thank you for your attention!