Recent Results on Key-Length Extension

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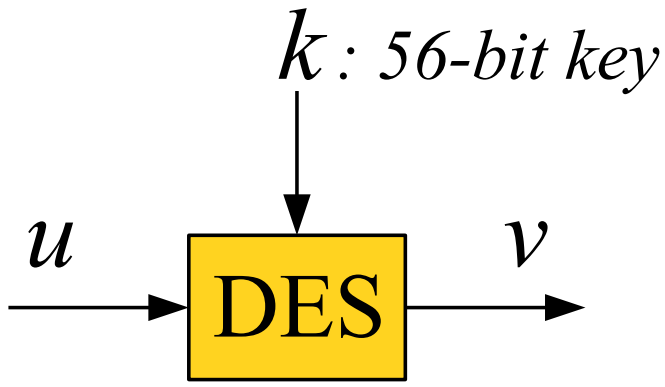
Content

- Introduction to key length extension
- Security proof of cascade encryption (Eurocrypt 2013)
- Recent results on key length extension schemes

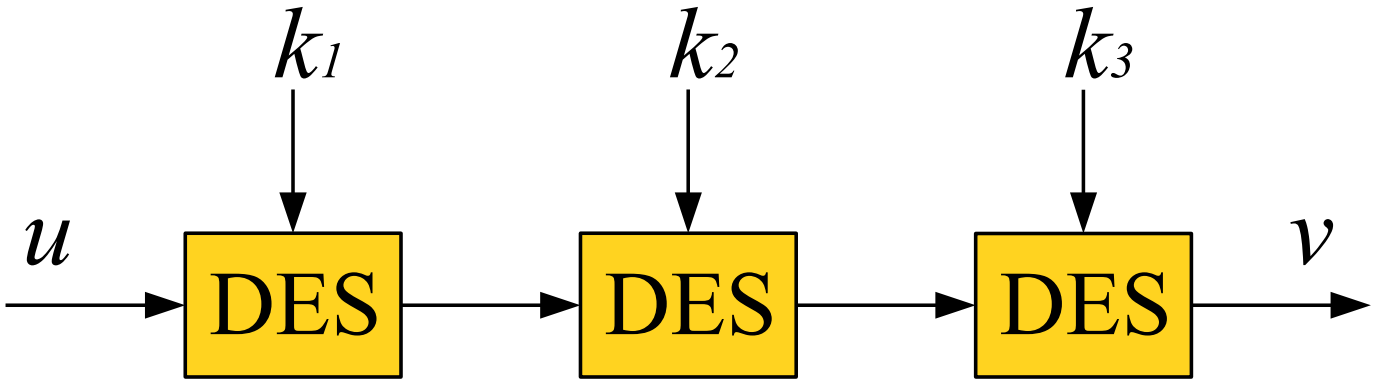
Blockciphers Using Short Keys

DES



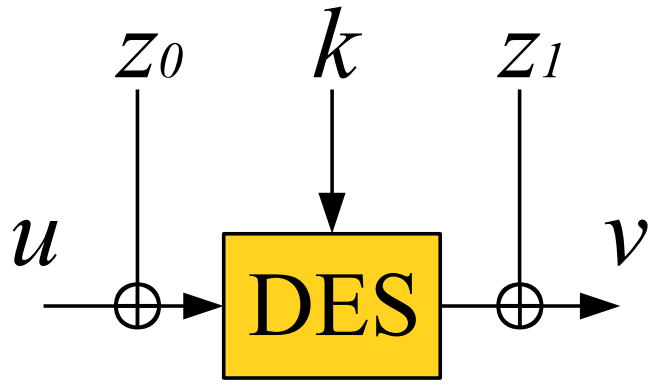
- Widely-used blockcipher using 56-bit keys
- No feasible attack faster than key exhaustive search
- Advances in computational power made key exhaustive search itself practical
  - Replaced by AES
  - Construction of DES-based encryption schemes employing longer keys: key-length extension

Triple-DES



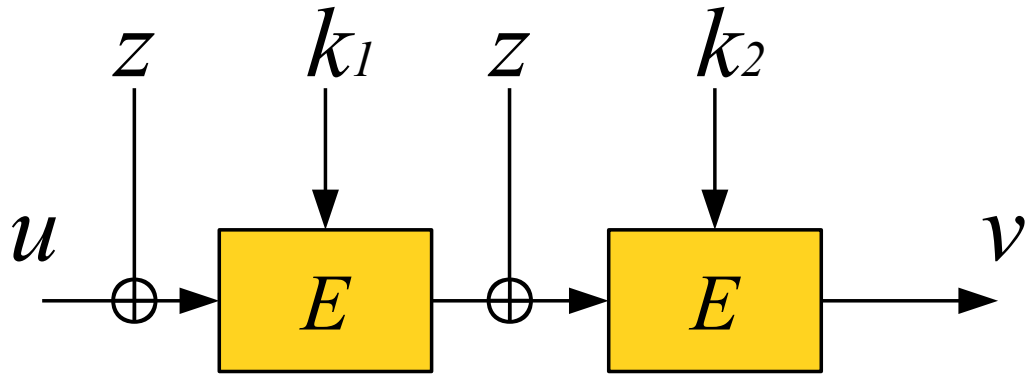
- Double-DES is vulnerable to a meet-in-the-middle attack
- Security proved up to $2^{\kappa+\frac{\min\{n,\kappa\}}{2}}$ queries
  - Bellare and Rogaway (Eurocrypt 2006)
  - Gaži and Maurer (Asiacrypt 2009): some flaws fixed

DESX



- Pre/post whitening keys used
- Security proved up to $2^{\frac{\kappa+n}{2}}$ queries
  - Kilian and Rogaway (Journal of Cryptology, 2001)

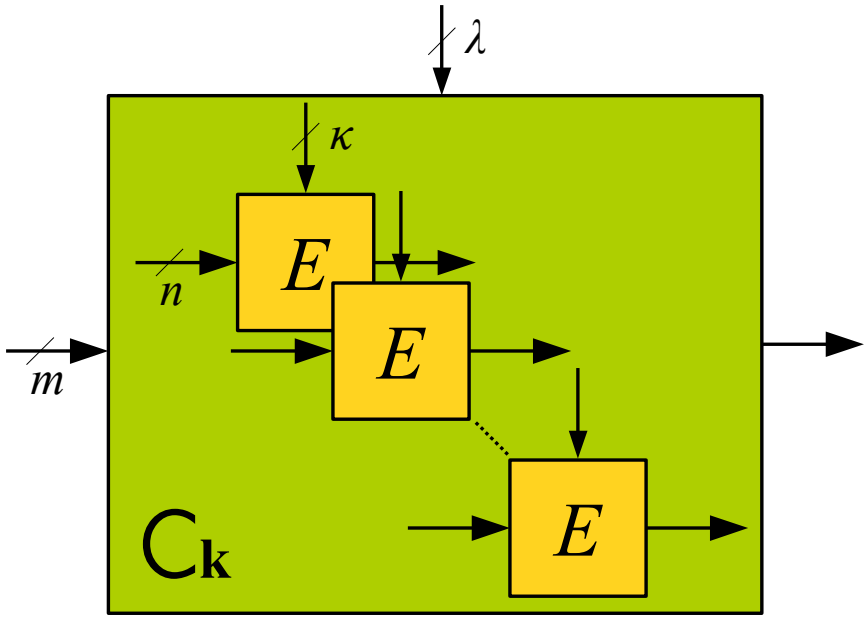
Randomized Cascade



- Cascade of DESX with some modification
- Security proved up to $2^{\kappa+\frac{n}{2}}$ queries
  - Gaži and Tessaro (Eurocrypt 2012)

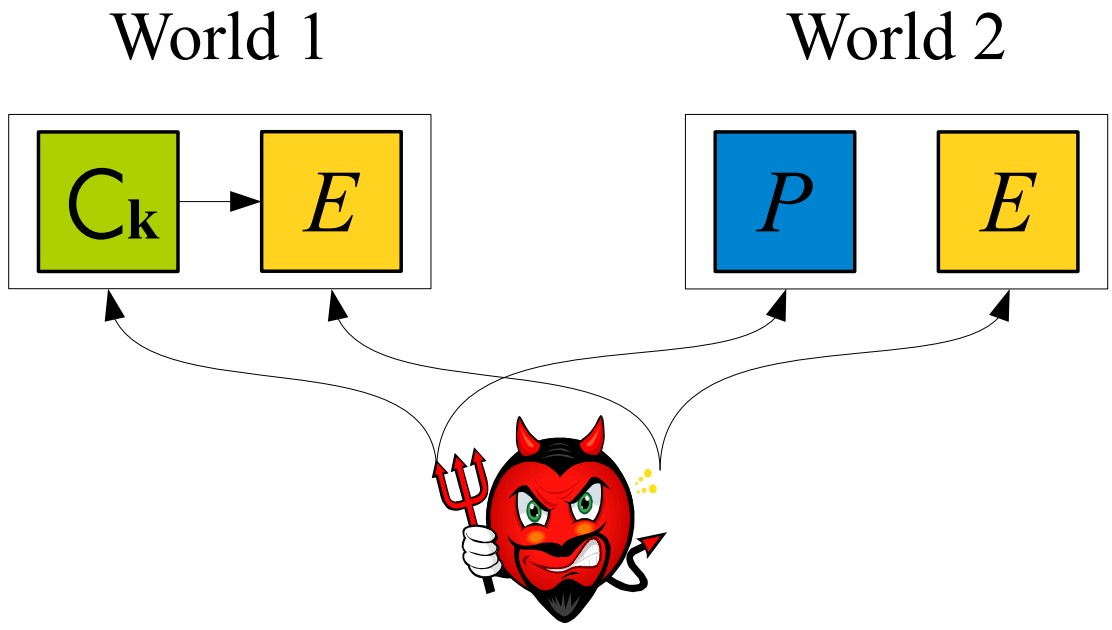
Key Length Extension

A λ-bit key m-bit encryption scheme C



- Makes a fixed number of calls to the underlying κ-bit key n-bit blockcipher E ($\lambda > \kappa$)
- Each key $\mathbf{k} = \{0,1\}^{\lambda}$ defines a permutation on $\{0,1\}^{m}$

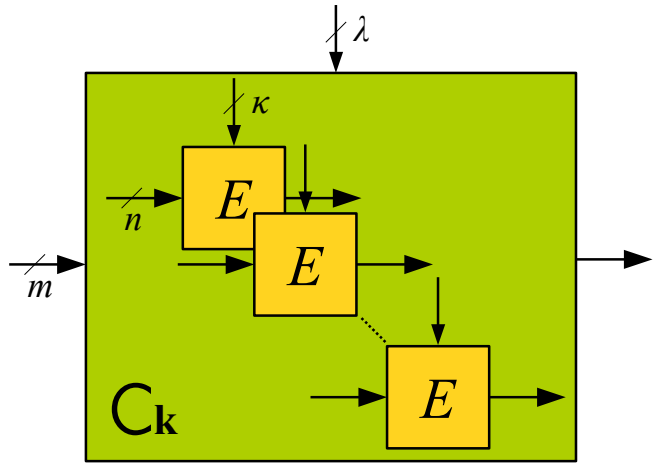
Security of Key Length Extension



- A distinguisher $\mathcal{A}$ wants to tell apart $(\mathsf{C}_{\mathbf{k}}[E], E)$ and (P, E)
  - by adaptively making forward and backward queries to the permutation and the blockcipher

$$\mathbf{Adv}^{\mathrm{PRP}}_{\mathsf{C}}(\mathcal{A}) = \mathbf{Pr}\left[P \xleftarrow{\$} \mathcal{P}_n, E \xleftarrow{\$} BC(\kappa, n) : \mathcal{A}[P, E] = 1\right]$$
$$- \mathbf{Pr}\left[\mathbf{k} \xleftarrow{\$} \{0,1\}^{\lambda}, E \xleftarrow{\$} BC(\kappa, n) : \mathcal{A}[\mathsf{C}_{\mathbf{k}}[E], E] = 1\right]$$

Bruce-force Attack of $2^{\kappa+n}$ Queries

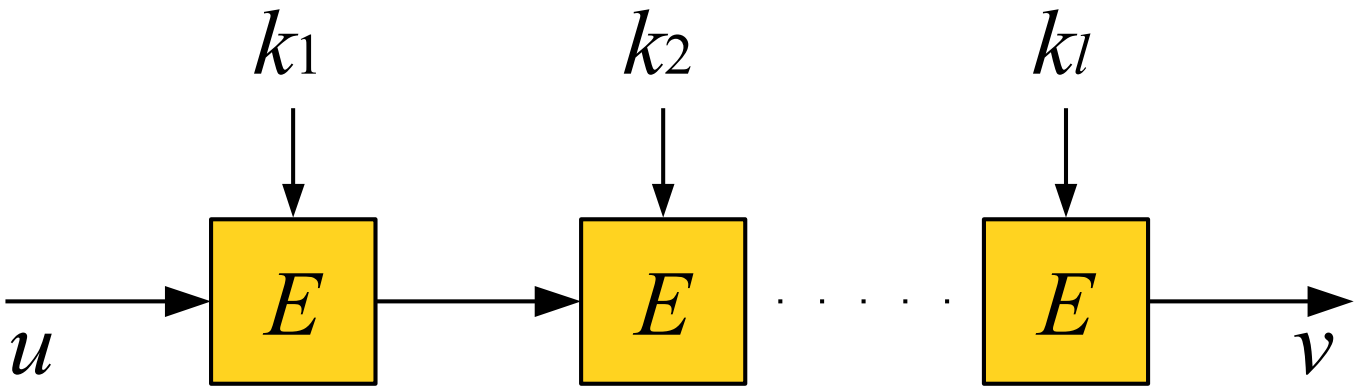


1. $\mathcal{A}$ makes all possible $2^{\kappa+n}$ queries to E.
2. $\mathcal{A}$ makes t nonadaptive forward queries to the outer permutation, recording query history $\mathcal{Q} = (u^i, v^i)_{1 \le i \le t}$.
3. If there is a λ-bit key $\mathbf{k}$ such that $\mathsf{C}_{\mathbf{k}}[E](u^i) = v^i$ for every $i = 1, \ldots, t$, then $\mathcal{A}$ outputs 0. Otherwise, $\mathcal{A}$ outputs 1.

$\mathbf{Adv}^{\mathrm{PRP}}_{\mathsf{C}}(\mathcal{A}) \approx 1$ as $t \gg \frac{\lambda}{m}$.

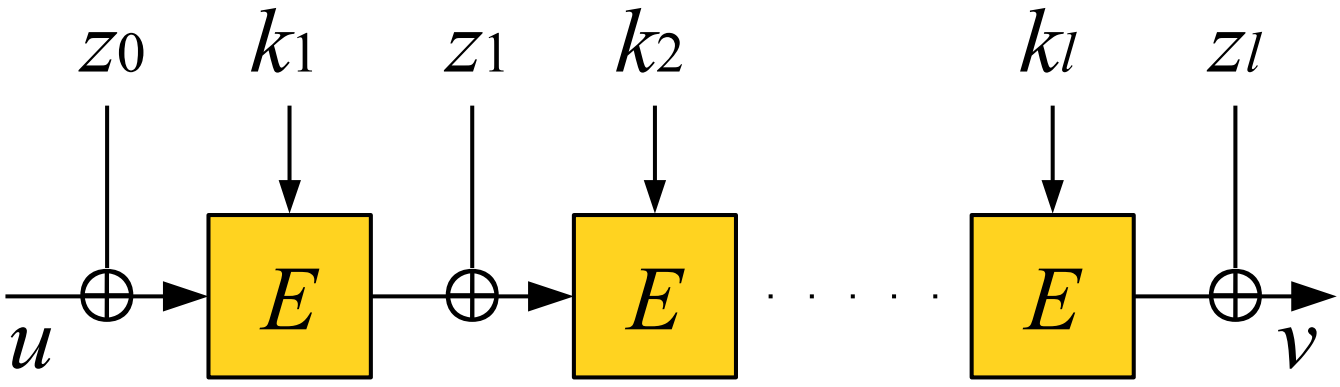
Key length extension with optimal security?

Cascade Encryption CE^l



- Security asymptotically proved up to $2^{\kappa+\min\{\frac{n}{2},\kappa\}}$ queries
  - Gaži and Maurer (Asiacrypt 2009)
- Proved up to $2^{\kappa+\min\{\kappa,n\}-\frac{16}{l}\left(\frac{n}{2}+2\right)}$ query complexity
  - Lee (Eurocrypt 2013)
  - Close to $2^{\kappa+\min\{\kappa,n\}}$ when the cascade length l is large
  - Asymptotically optimal if $n \leq \kappa$

Xor-cascade Encryption XCE^l



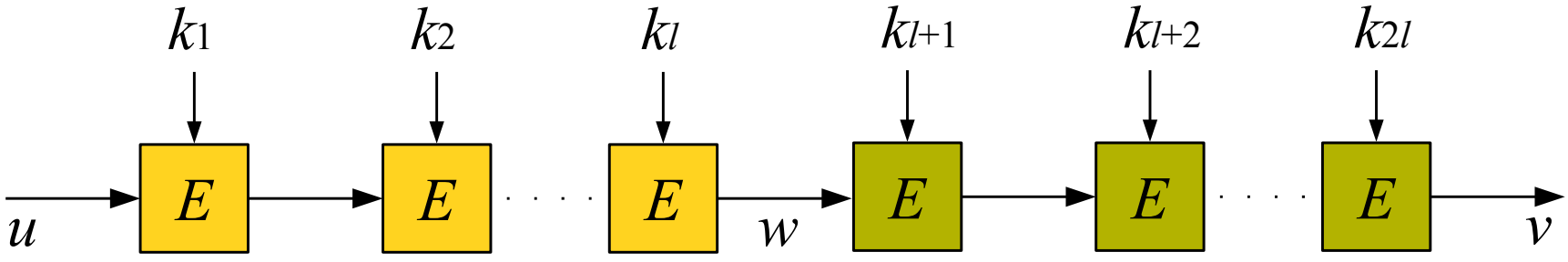
- Security proved up to $2^{\kappa+n-\frac{8}{l}\left(\frac{n}{2}+2\right)}$ query complexity
  - Lee (Eurocrypt 2013)
  - Close to $2^{\kappa+n}$ when the cascade length l is large
- Gazi improved on this bound (Crypto 2013)

Security Proof of 2*l*-cascade Encryption

Proof Strategy

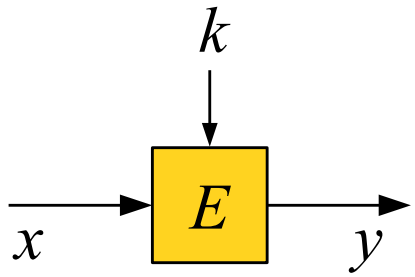
1. Prove NCPA-security of *l*-cascade encryption
2. Lift NCPA-security to CCA-security by composing two independent components
   - Mauer, Pietrzak and Renner's framework (Crypto 2007)
   - Combinatorial interpretation

"Random key space separation" technique needed

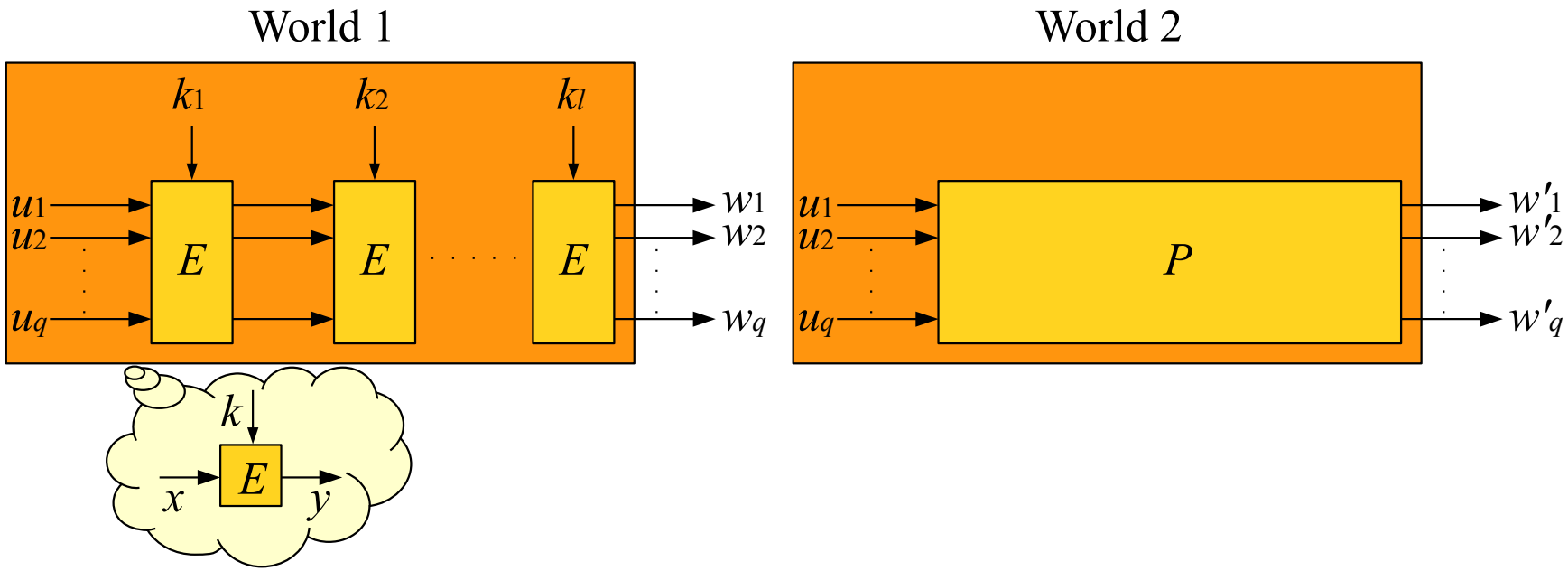


NCPA Adversary

1. Makes q queries to the underlying blockcipher

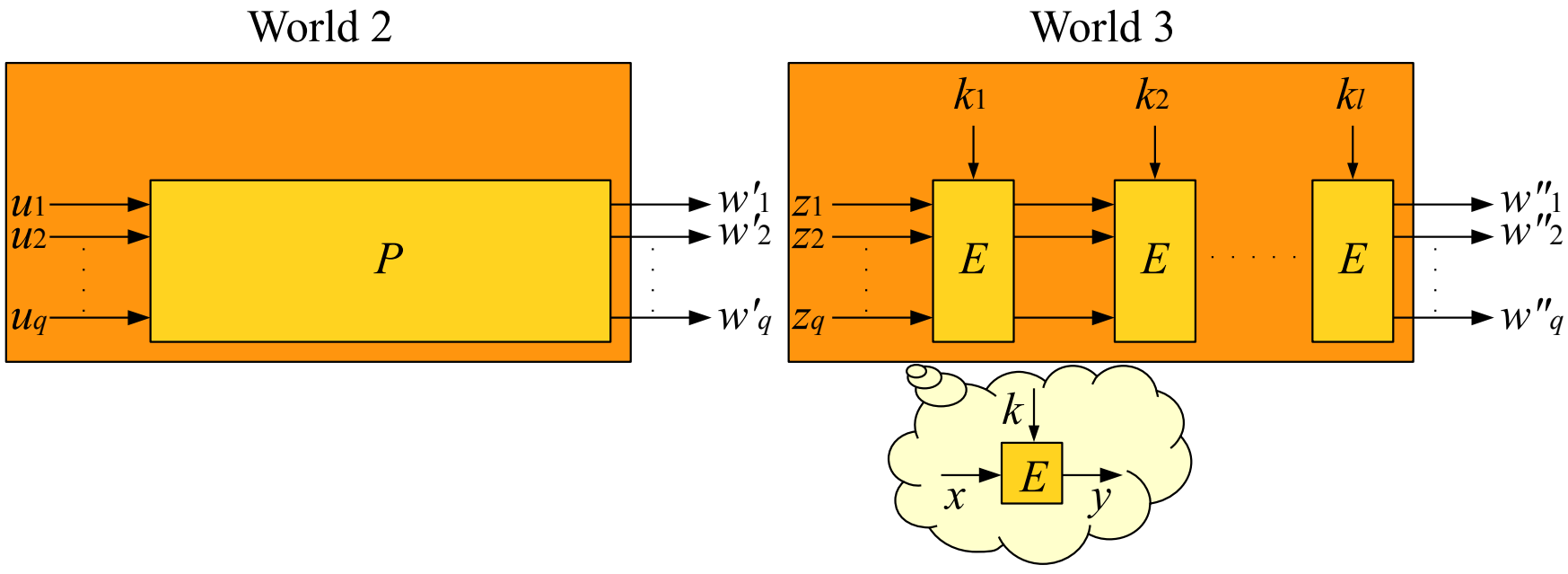


2. Determine q queries $u_1, \ldots, u_q$ to the outer permutation and distinguish two worlds:



Same Construction, Different Inputs

For distinct random inputs $z_1, \ldots, z_q$, World 2 and World 3 are exactly the same



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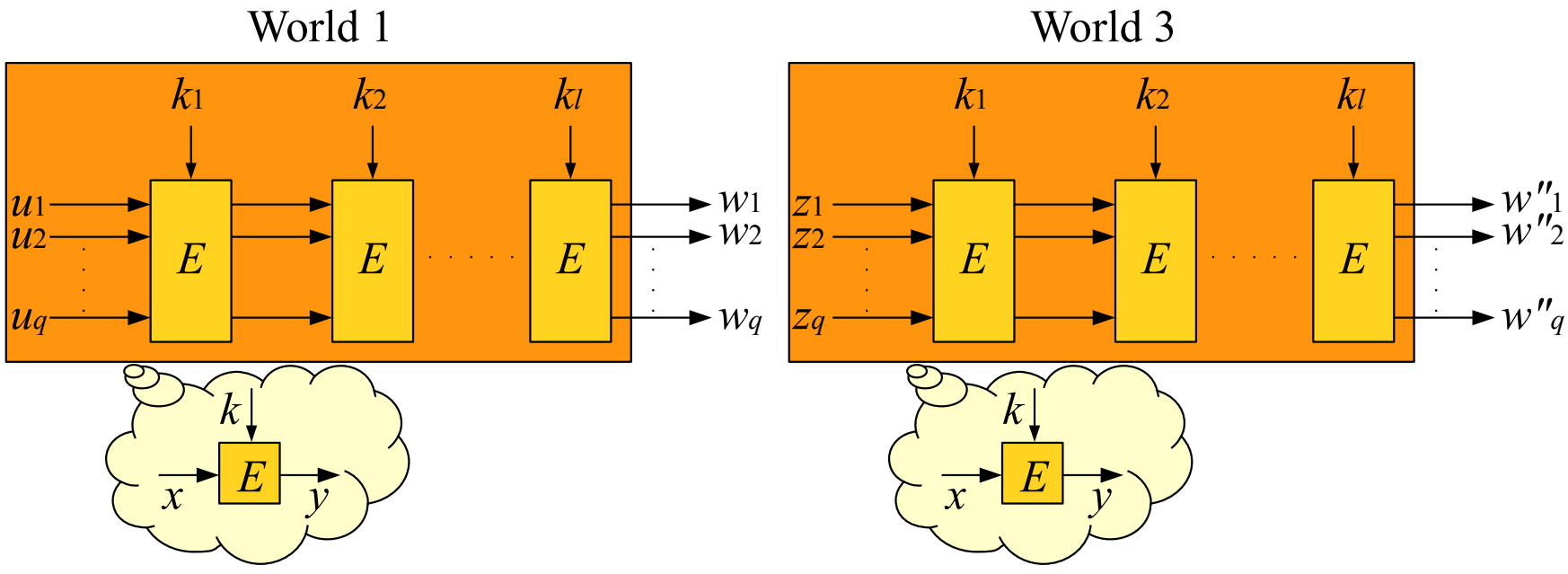

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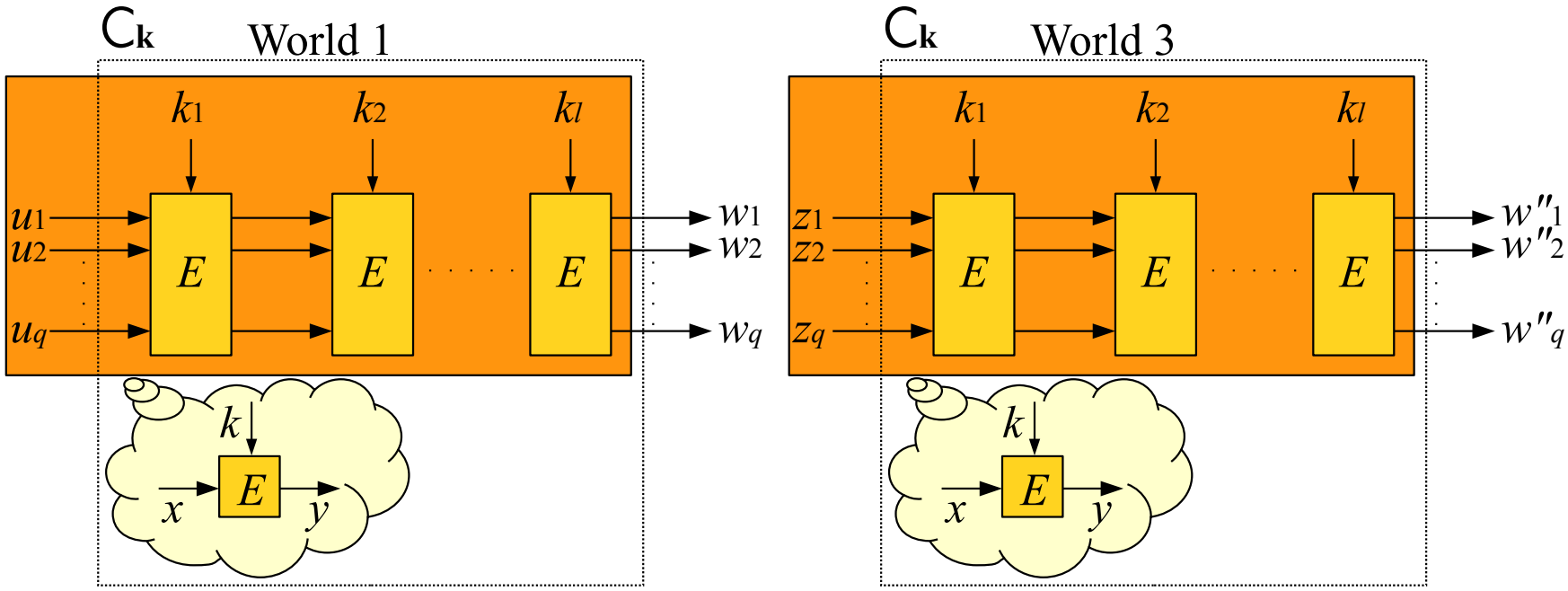
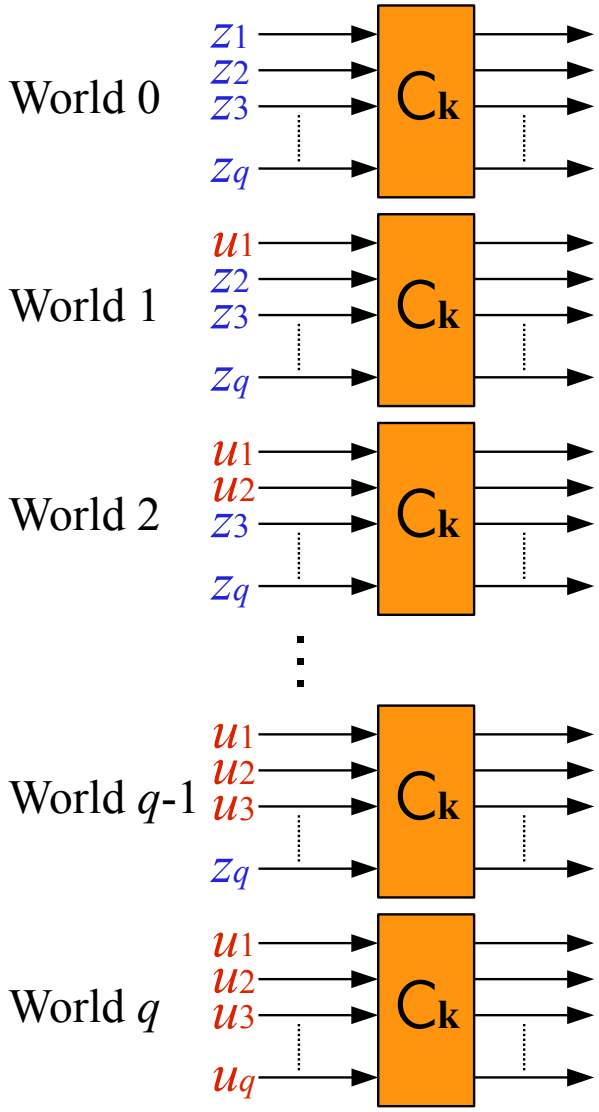
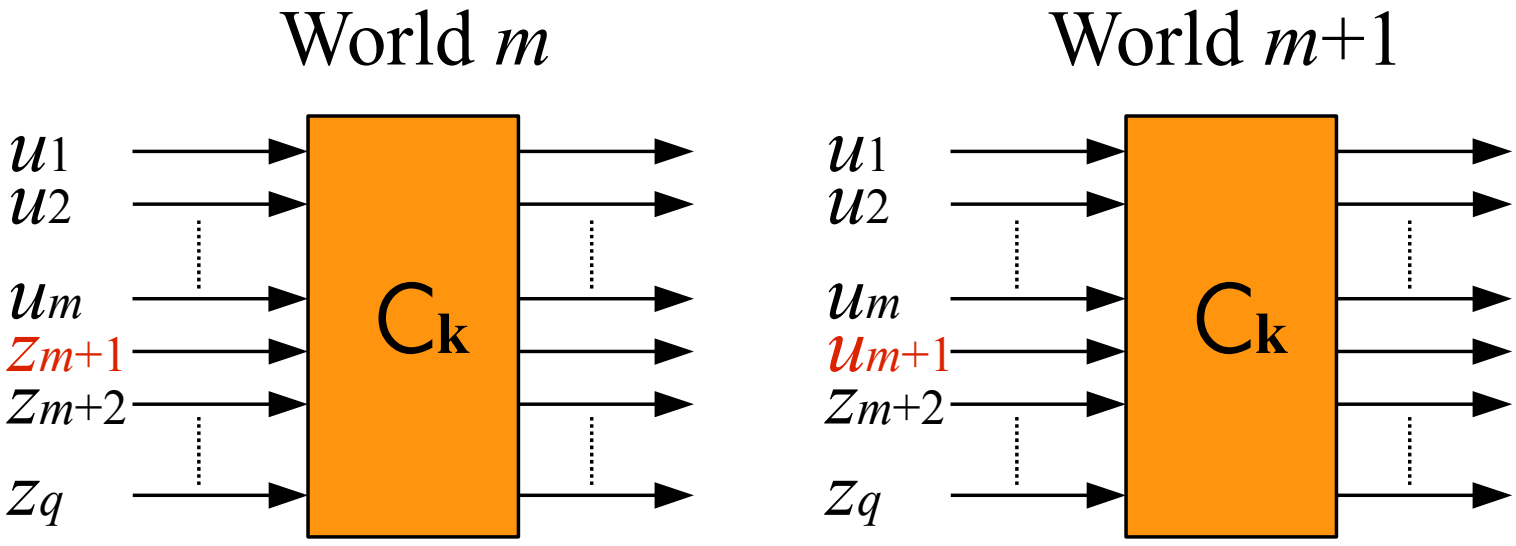


Hybrid Argument

Input values change one by one

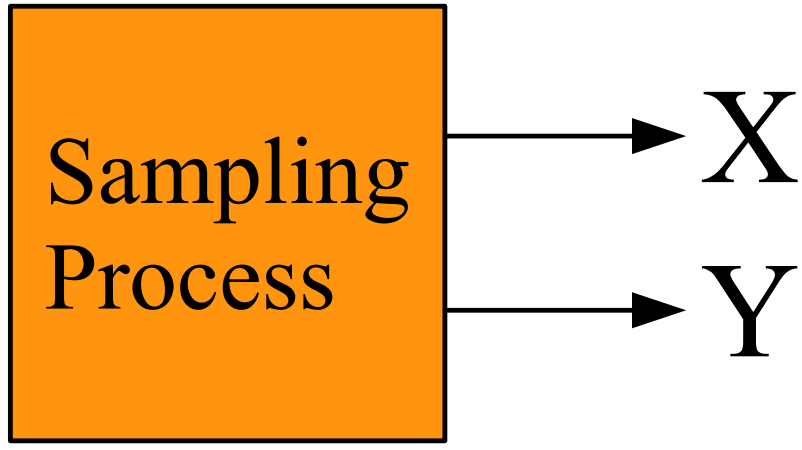


Distinguishing World m and World $m+1$



- For two probability distributions of the outputs(q-tuples), we will upper bound their statistical distance

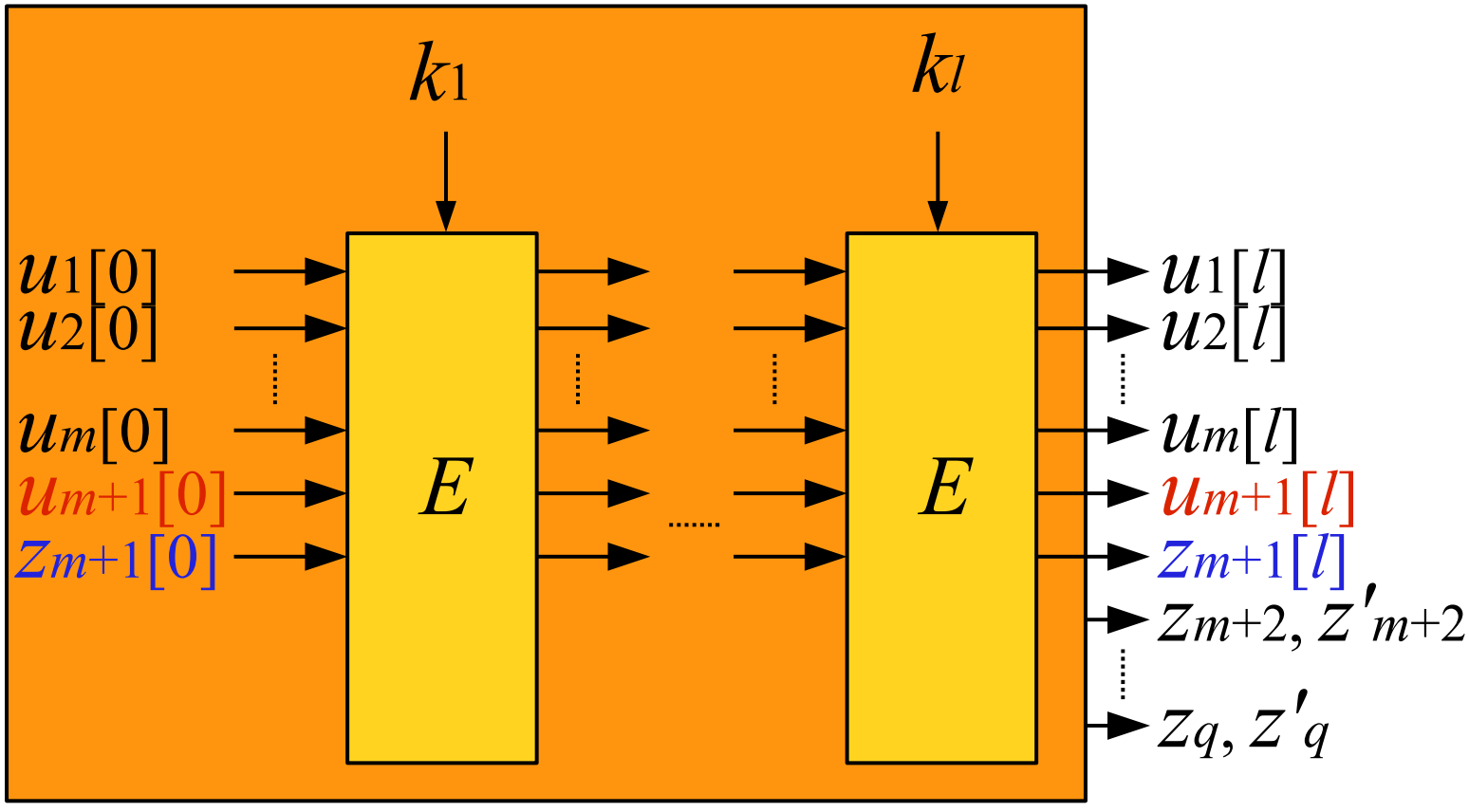
Coupling Technique



If $X \sim \mu$ and $Y \sim \nu$, then $\|\mu - \nu\| \leq \mathbf{Pr}[X \neq Y]$

Need to carefully design the sampling process such that $X \sim \mu$ and $Y \sim \nu$ (called a "coupling") and $\mathbf{Pr}[X \neq Y]$ is small

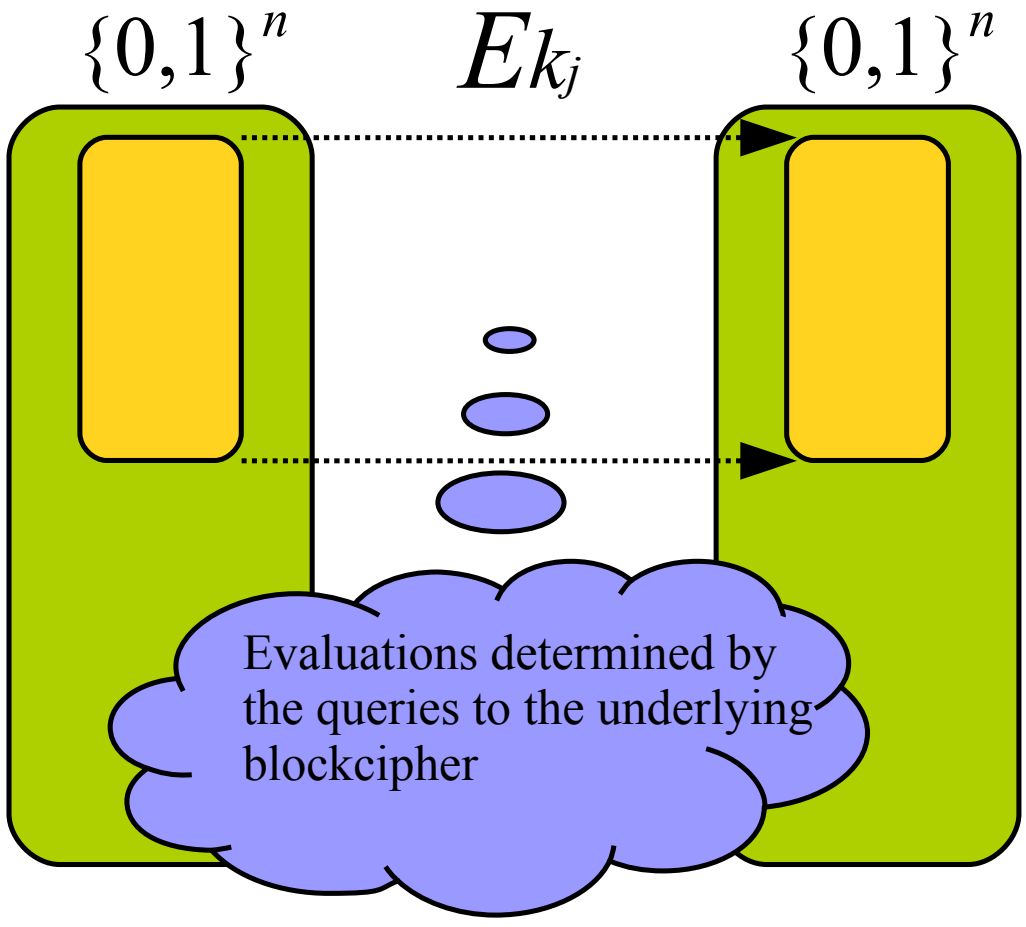
How to Couple World m and World $m+1$



$$X=(u_1[l],\ldots, u_m[l], u_{m+1}[l], z_{m+2},\ldots, z_q)$$

$$Y=(u_1[l],\ldots, u_m[l], z_{m+1}[l], z'_{m+2},\ldots, z'_q)$$

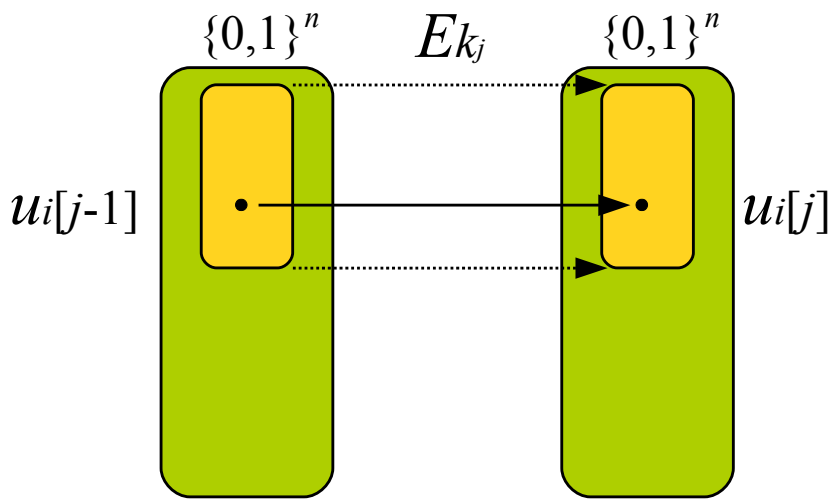
Update of $u_1[j-1], \ldots, u_m[j-1]$ at the j-th Round



Update of $u_1[j-1], \ldots, u_m[j-1]$ at the j-th Round

For $i = 1, \ldots, m$:

- $u_i[j-1] \in \mathrm{Dom}(E_{k_j})$



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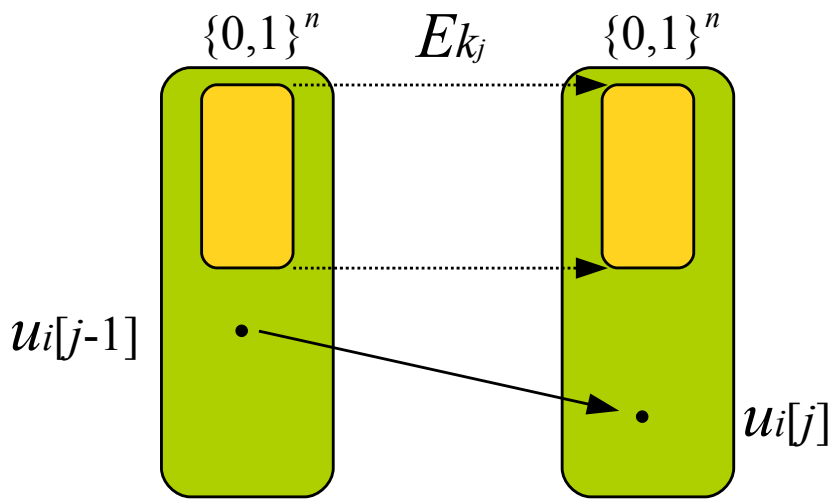
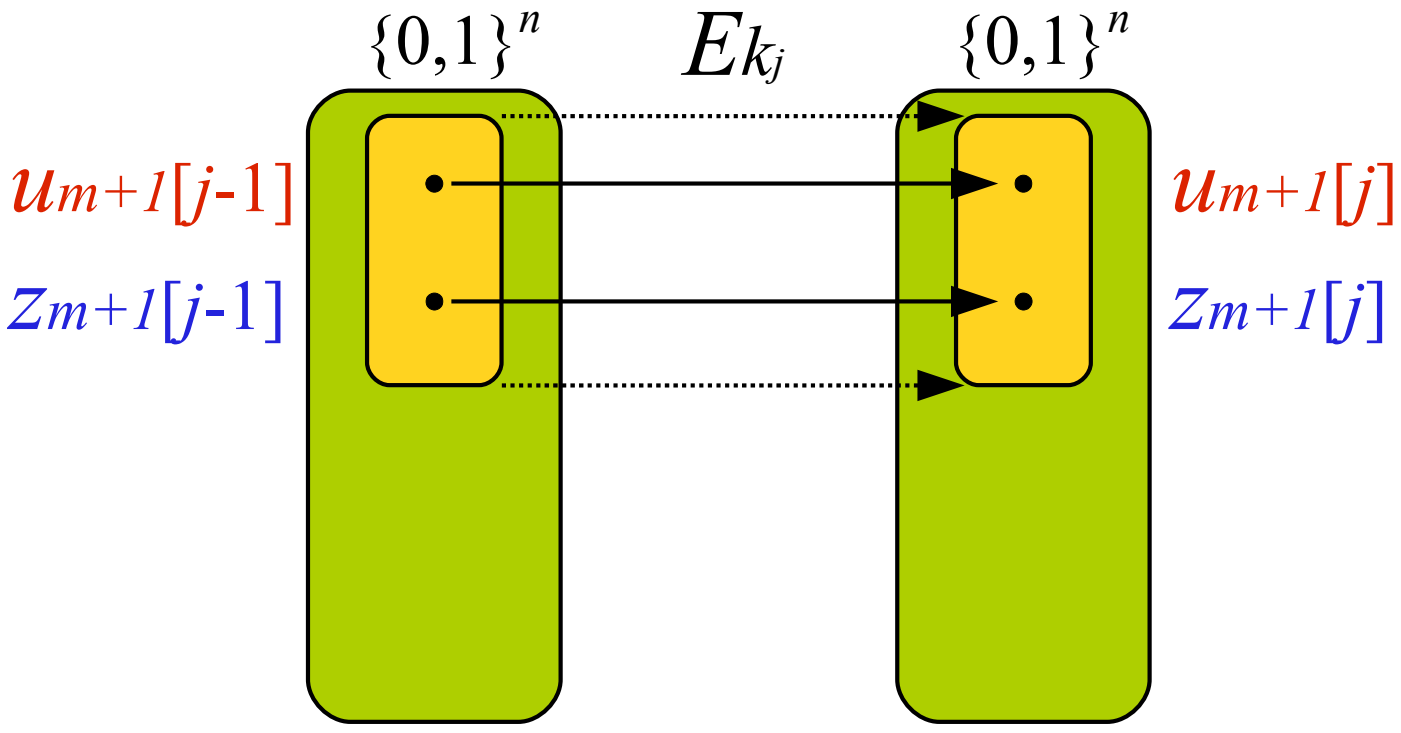


Update of $u_{m+1}[j-1]$ and $z_{m+1}[j-1]$ at the j-th Round

$u_{m+1}[j-1] \in \mathrm{Dom}(E_{k_j})$ and $z_{m+1}[j-1] \in \mathrm{Dom}(E_{k_j})$



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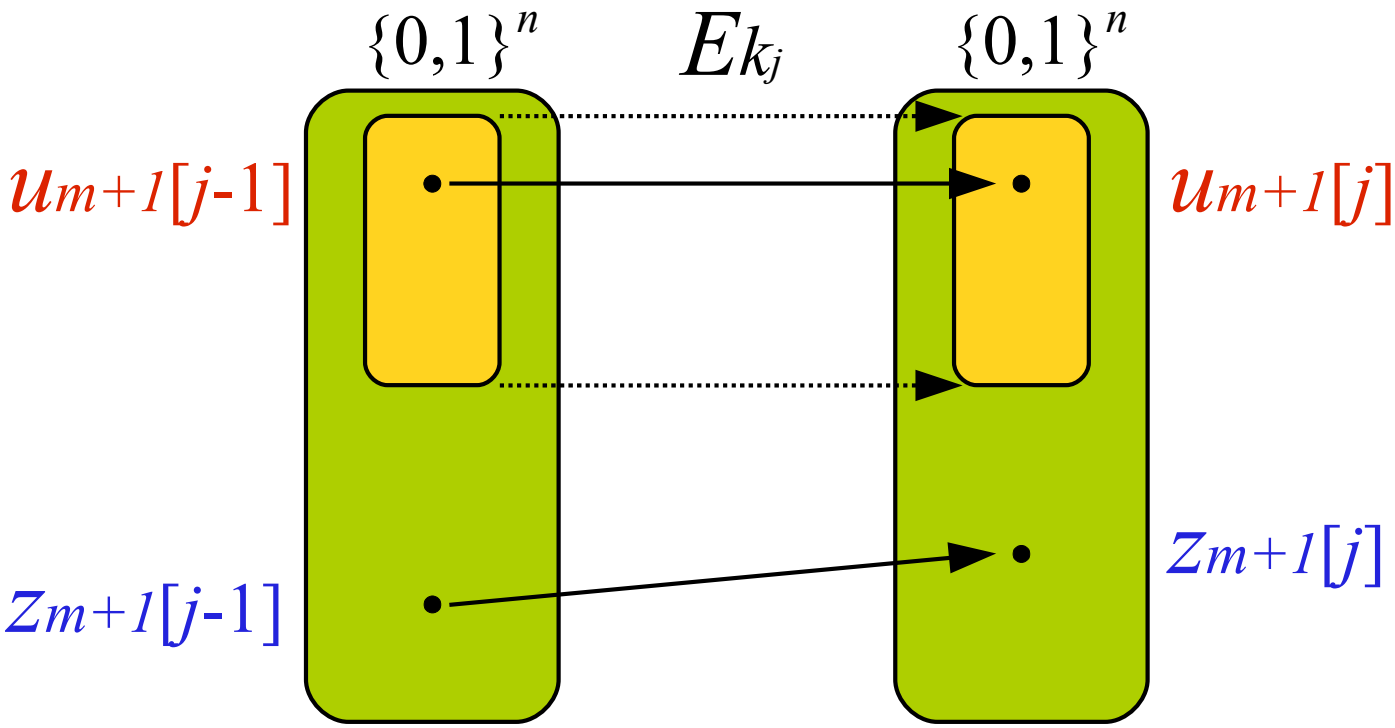

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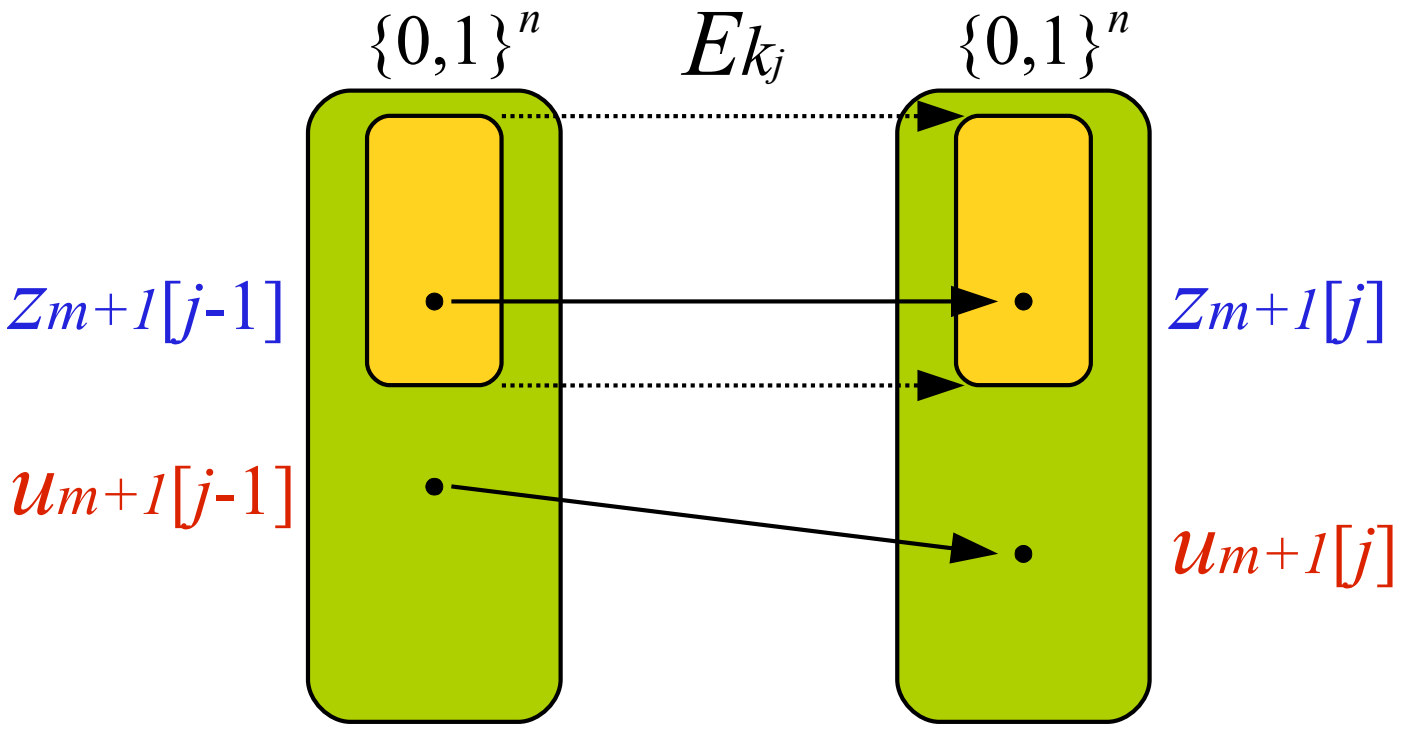

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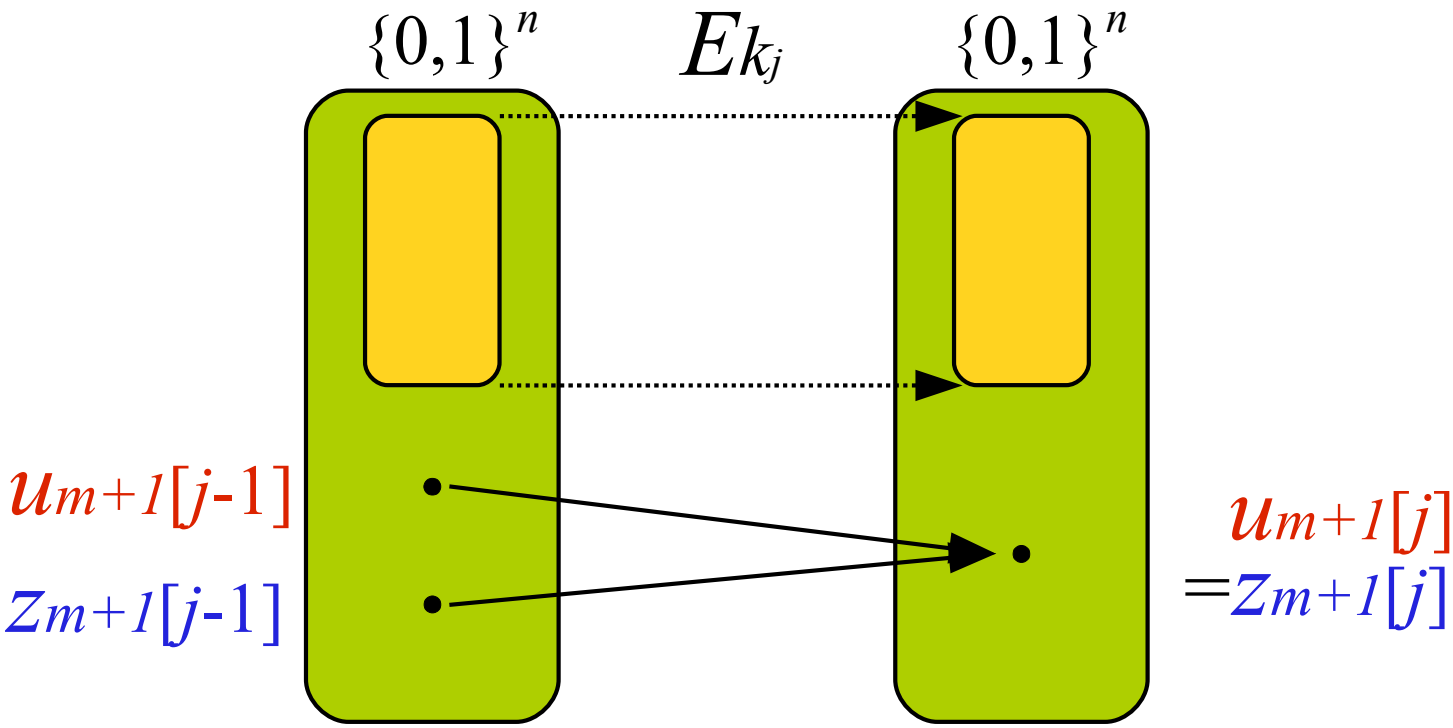


Defining $z_{m+2}, \ldots, z_q$ and $z'_{m+2}, \ldots, z'_q$

If $u_{m+1}[l] \neq z_{m+1}[l]$

$$\text{Distinct } z_{m+2}, \ldots, z_q \xleftarrow{\$} \{0,1\}^n \backslash \{u^1[l], \ldots, u^m[l], u^{m+1}[l]\}$$
$$\text{Distinct } z'_{m+2}, \ldots, z'_q \xleftarrow{\$} \{0,1\}^n \backslash \{u^1[l], \ldots, u^m[l], z^{m+1}[l]\}$$

If $u_{m+1}[l] = z_{m+1}[l]$

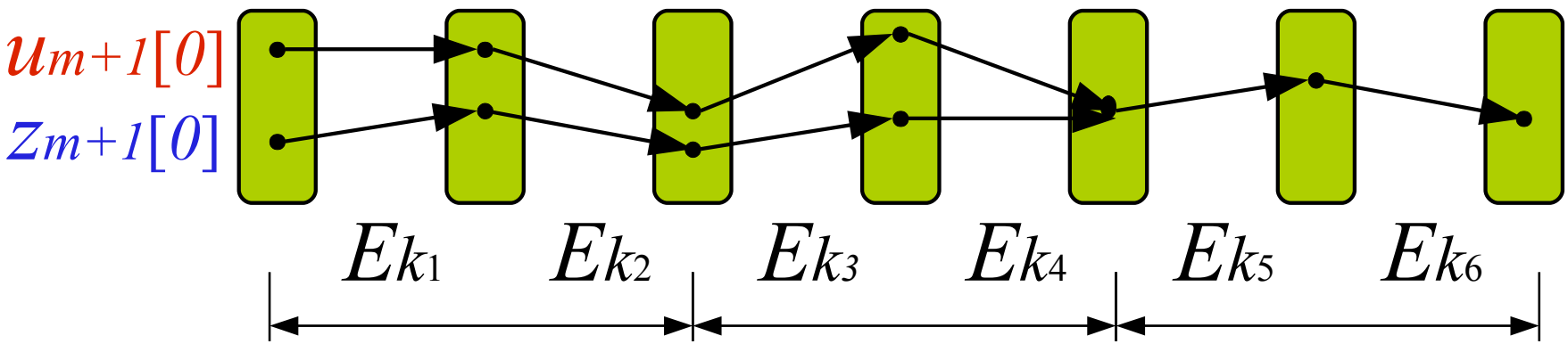
$$\text{Distinct } z_{m+2}, \ldots, z_q \xleftarrow{\$} \{0,1\}^n \backslash \{u^1[l], \ldots, u^m[l], u^{m+1}[l]\}$$
$$(z'_{m+2}, \ldots, z'_q) \leftarrow (z_{m+2}, \ldots, z_q)$$

X and Y sample the outputs of World m and World $m+1$, respectively

Upper Bounding $\Pr[X \neq Y]$

$$\Pr[X \neq Y] = \Pr[u_{m+1}[l] \neq z_{m+1}[l]]$$

$$\leq \prod_{h=1}^{\frac{l}{2}} \Pr\left[u_{m+1}[2h] \neq z_{m+1}[2h] \;\middle|\; u_{m+1}[2h-2] \neq z_{m+1}[2h-2]\right]$$



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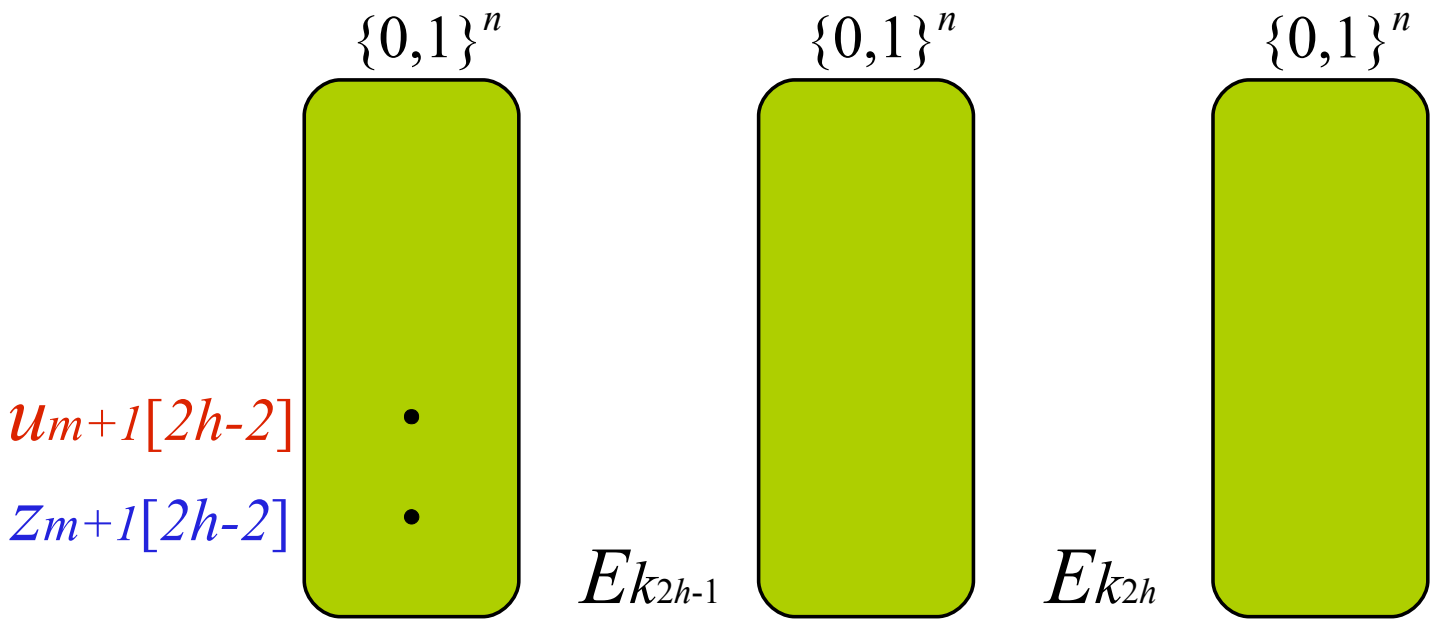
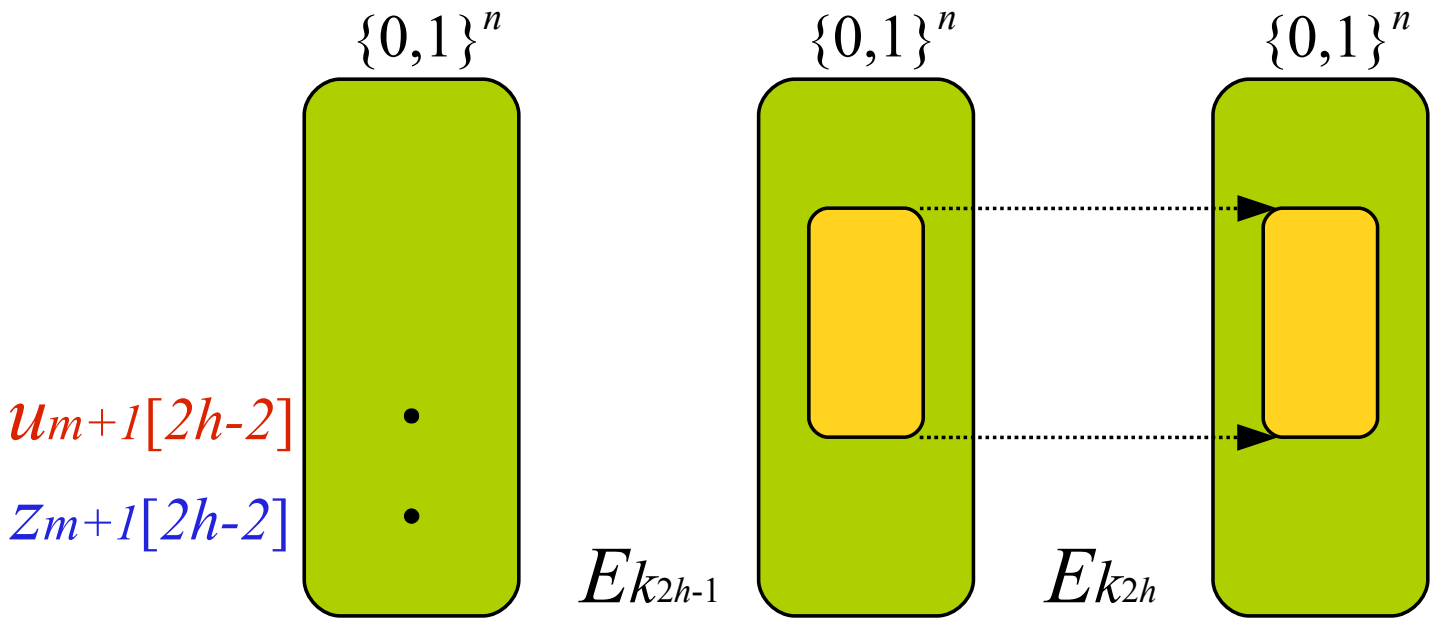


Upper Bounding

$$\Pr\left[u_{m+1}[2h] \neq z_{m+1}[2h] \;\middle|\; u_{m+1}[2h-2] \neq z_{m+1}[2h-2]\right]$$

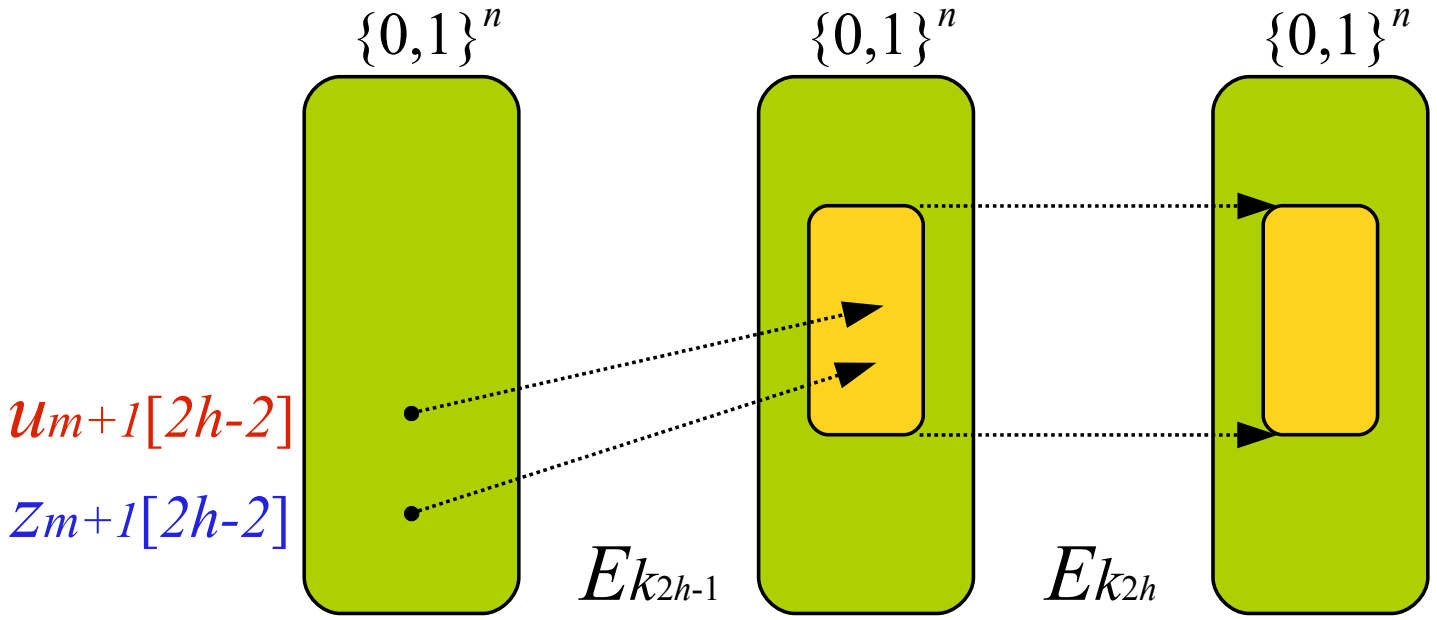
- The size of $\mathrm{Dom}(E_{k_{2h-1}})$ and $\mathrm{Dom}(E_{k_{2h}}) \leq M$
  - except with probability $\frac{2q}{M2^{\kappa}}$



Upper Bounding

$$\Pr\left[u_{m+1}[2h] \neq z_{m+1}[2h] \;\middle|\; u_{m+1}[2h-2] \neq z_{m+1}[2h-2]\right]$$

- Upper bound the probability that one of $u_{m+1}[2h-2]$ and $z_{m+1}[2h-2]$ maps into $\mathrm{Dom}(E_{k_{2h}})$
  - By choosing key k_{2h-1}: probability $\frac{2M\beta}{2^\kappa}$ with a parameter β
  - By random sampling: probability $\frac{2M}{N}$



Upper Bounding

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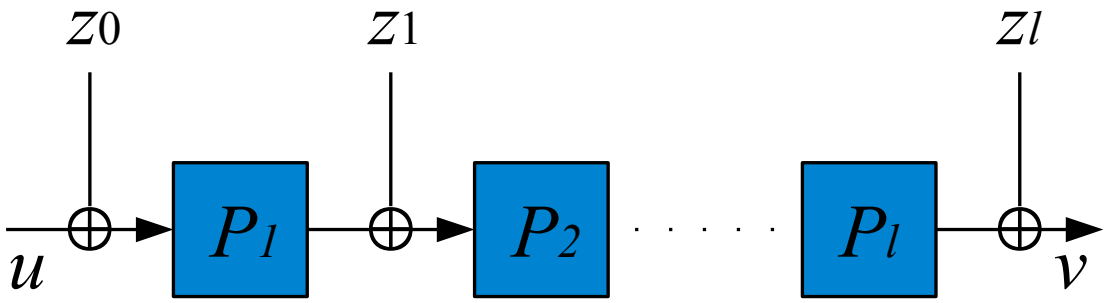
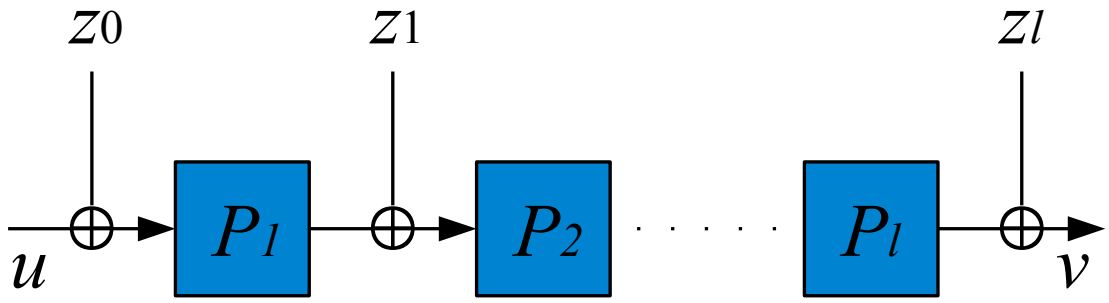
  - By choosing key k_{2h-1}: probability $\frac{2M\beta}{2^{\kappa}}$ with a parameter β
  - By random sampling: probability $\frac{2M}{N}$
- $\Pr[X \neq Y] \leq \left(\frac{2q}{M2^{\kappa}} + \frac{2M\beta}{2^{\kappa}} + \frac{2M}{N}\right)^{\frac{l}{2}}$
- Optimize the parameters β and M to obtain the result

Gazi's Result (Crypto 2013)



- Generic Attacks
  - Generic attacks on CE^l with $2^{\kappa+\frac{l-1}{l+1}n}$ (resp. $2^{\kappa+\frac{l-2}{l}n}$) queries for odd (resp. even) length l
  - Generic attacks on XCE^l with $2^{\kappa+\frac{l-1}{l}n}$ queries
- Security Proof
  - The security of XCE^l $=$ the security of a key-alternating ciphers of length $l-1$ + key length κ
  - XCE^l is secure up to $2^{\kappa+\frac{l-1}{l+1}n}$ (resp. $2^{\kappa+\frac{l-2}{l}n}$) query complexity for odd (resp. even) length l

Chen and Steinberger's Result (Eprint Archive)



- Proved a key-alternating cipher of length l is secure up $2^{\frac{l}{l+1}n}$ queries
- Implies XCE^l is secure up to $2^{\kappa+\frac{l-1}{l}n}$ queries
- Closed the security problem of XCE^l
- What about CE^l?

Thank You