Pseudo-cryptanalysis of the Original BMW

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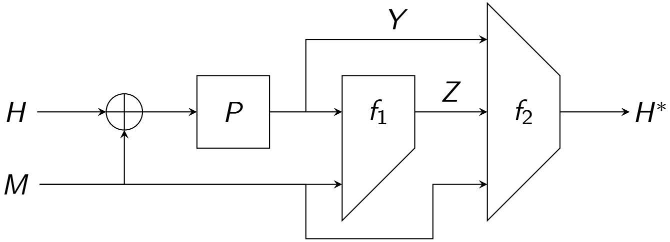
CCRG Seminar, NTU
18 January, 2010

Blue Midnight Wish

- Developed by Gligoroski et al.
- Four variants (224-, 256-, 384-, 512-bit)
- In the second round of the SHA-3 competition
- Was tweaked between first and second round
- My results are on the first version!

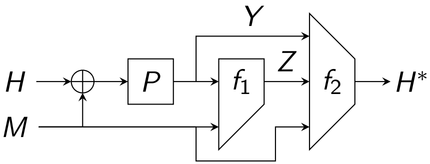
High-level design of the compression function



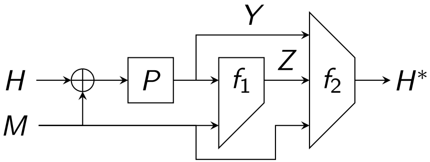
- H, M, Y, Z, H^*: 16 words each (e.g.: $H_0, \ldots, H_{15}$)
- Word size 32/64 (BMW-256/BMW-512).

The permutation P

- Easy to invert
- Given M and Y, compute $H = P^{-1}(Y) \oplus M$
- Details of P irrelevant here.



The function f_1



- Multipermutation
  - $f_1(Y, \cdot)$ a permutation
  - $f_1(\cdot, M)$ a permutation
- Permutations are invertible
- "Simple" and "complex" rounds (security parameter).

Example: a complex round

Let $Q = Y \| Z$, with Z initially null.

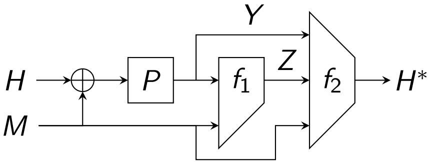
$$
\begin{aligned}
Q_{i+16} \quad \leftarrow \quad & s_1(Q_i) + s_2(Q_{i+1}) + s_3(Q_{i+2}) + s_0(Q_{i+3}) + \\
& s_1(Q_{i+4}) + s_2(Q_{i+5}) + s_3(Q_{i+6}) + s_0(Q_{i+7}) + \\
& s_1(Q_{i+8}) + s_2(Q_{i+9}) + s_3(Q_{i+10}) + s_0(Q_{i+11}) + \\
& s_1(Q_{i+12}) + s_2(Q_{i+13}) + s_3(Q_{i+14}) + s_0(Q_{i+15}) + \\
& \underbrace{M_i + M_{i+3} - M_{i+10}}_{W_i} + K_i,
\end{aligned}
$$

- Mapping from M to W corresponds to invertible matrix multiplication: $W = \mathbf{B} \cdot M$

The function f_2

- Details later

Preimages – idea of the attack



- Force $Z = 0$
- Now f_2 is *very* simple.

f_2 with $Z = 0$

$$
\begin{aligned}
H_0^* &= M_0 + Y_0 \\
&\vdots \\
H_7^* &= M_7 + Y_7 \\
H_8^* &= (M_4 + Y_4)^{\lll 9} + M_8 + Y_8 \\
H_9^* &= (M_5 + Y_5)^{\lll 10} + M_9 + Y_9 \\
H_{10}^* &= (M_6 + Y_6)^{\lll 11} + M_{10} + Y_{10} \\
H_{11}^* &= (M_7 + Y_7)^{\lll 12} + M_{11} + Y_{11} \\
H_{12}^* &= (M_0 + Y_0)^{\lll 13} + M_{12} + Y_{12} \\
H_{13}^* &= (M_1 + Y_1)^{\lll 14} + M_{13} + Y_{13} \\
H_{14}^* &= (M_2 + Y_2)^{\lll 15} + M_{14} + Y_{14} \\
H_{15}^* &= (M_3 + Y_3)^{\lll 16} + M_{15} + Y_{15}
\end{aligned}
$$

Inverting f_1

Remember: $Z = 0$

- After choosing W_{15}, we can compute Y_{15}
- ...or we can choose Y_{15} and compute W_{15}
- The same with W_{14}, W_{13}, ...

f_2 with $Z = 0$

$$
\begin{aligned}
H_0^* &= M_0 + Y_0 \\
&\vdots \\
H_7^* &= M_7 + Y_7 \\
H_8^* &= (M_4 + Y_4)^{\lll 9} + M_8 + Y_8 \\
H_9^* &= (M_5 + Y_5)^{\lll 10} + M_9 + Y_9 \\
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H_{14}^* &= (M_2 + Y_2)^{\lll 15} + M_{14} + Y_{14} \\
H_{15}^* &= (M_3 + Y_3)^{\lll 16} + M_{15} + Y_{15}
\end{aligned}
$$

Choosing words in M and W concurrently

- Consider the definition of W_{15}:

$$W_{15} = M_{15} + M_2 - M_9$$

- We can "free" W_{15}
- Example: Replace everywhere M_2 by

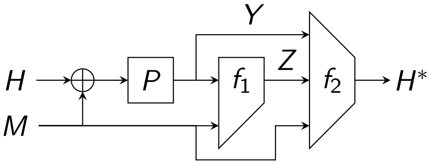
$$W_{15} - M_{15} + M_9$$

Controlling output words

- I.e., we can choose some words in M, and some words in W (at most 16 in total)
- Example: choose $Y_6, \ldots, Y_{15}$ and $M_6, M_7, M_{10}, M_{11}, M_{14}, M_{15}$
- Allows to control:

$$\begin{aligned}
H_6^* &= M_6 + Y_6 \\
H_7^* &= M_7 + Y_7 \\
H_{10}^* &= (M_6 + Y_6)^{\lll 11} + M_{10} + Y_{10} \\
H_{11}^* &= (M_7 + Y_7)^{\lll 12} + M_{11} + Y_{11}
\end{aligned}$$

Summary



- We can control up to four output words
- Complexity ~ 1 compression function evaluation
- Reduces complexity of preimage, second preimage, collision attacks on compression function
- Can be extended to pseudo-attacks.

Pseudo-attack complexities

Variant	Pseudo-collision	Pseudo-(second) preimage
BMW-224	2^{81} (2^{112})	2^{161} (2^{224})
BMW-256	2^{97} (2^{128})	2^{193} (2^{256})
BMW-384	2^{128} (2^{192})	2^{256} (2^{384})
BMW-512	2^{192} (2^{256})	2^{384} (2^{512})

Conclusion

- In the paper: near-collision attack in time $\sim 2^{15}$
- All results on Original BMW
- BMW tweaked – e.g., H now affects f_1 directly
- These attacks do not apply to Tweaked BMW

Thanks!